PiWorks Seminar

The day-to-day life of a low dimensional topologist

Ana G. Lecuona - Univ. of Glasgow - 2022

KNOTS: $k: S^1 \hookrightarrow \mathbb{R}^3$

"up to ambient isotopy"

In grl: $k: S^1 \hookrightarrow S^3 = \mathbb{R}^3 \cup \{\infty\}$

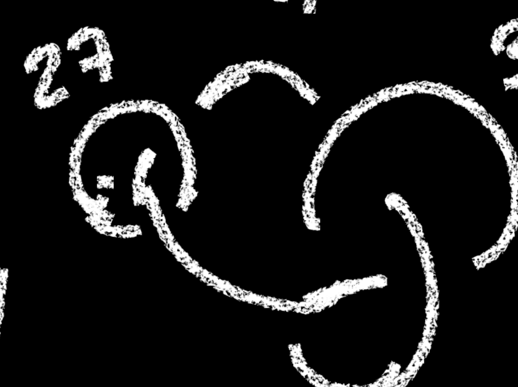
Relevance

- Applied: { DNA modelling, solar flares, dynamical vortices
- Pure: "low dim topology"

dim = 1, 2, 3, 4.

3-mnfld

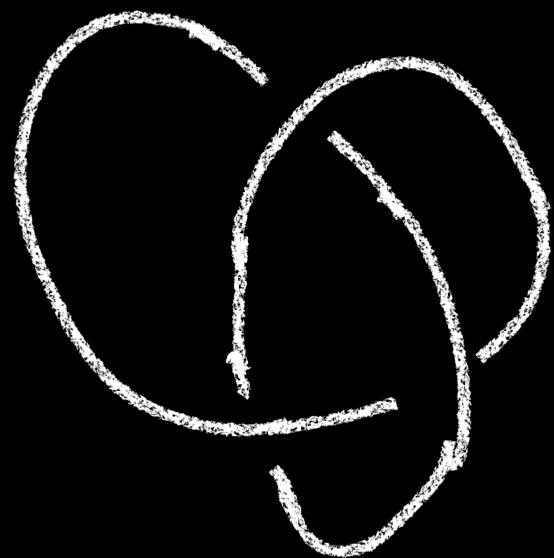
$\parallel$



$\parallel$

4-mnflds.

SURFACES BOUNDED BY KNOTS



non-orientable

"Seifert algorithm"

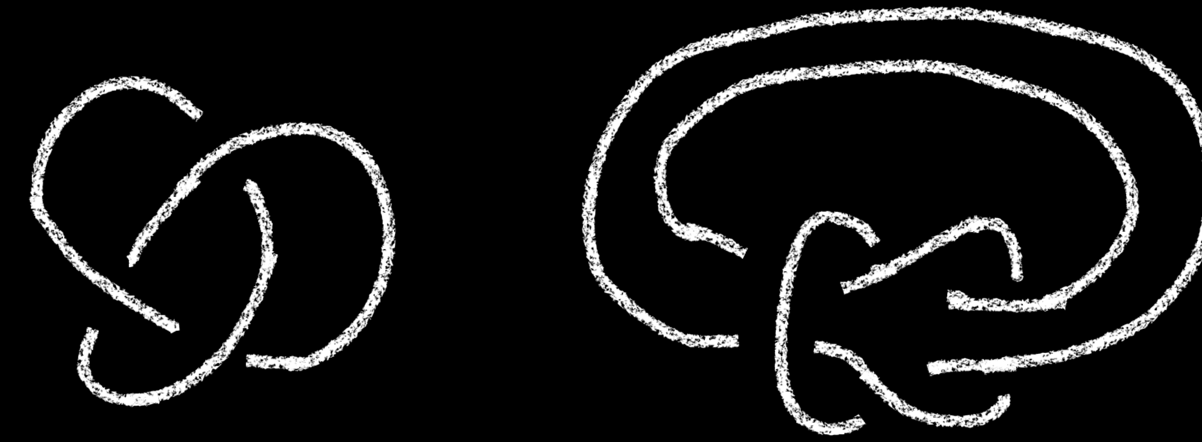
orientable

$\Delta_K(t)$ polynomial

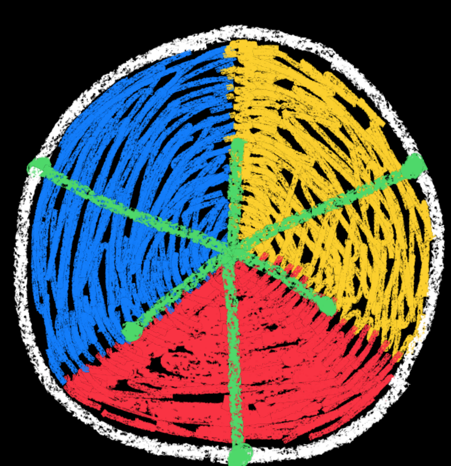
"$\Delta(t) = -t^{-1} + 1 - t$"

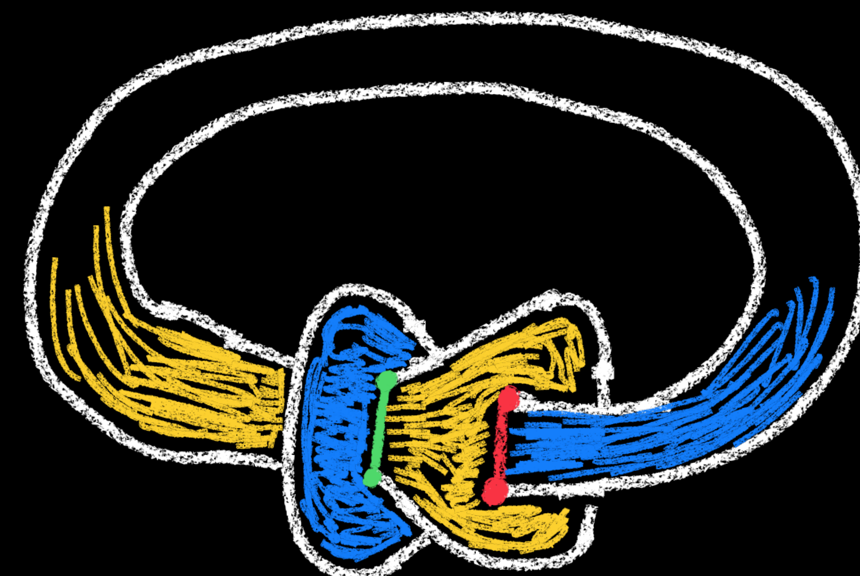
CLOSER LOOK TO INMERSED SURFACES

$$k: S^1 = \partial D^2 \hookrightarrow S^3$$

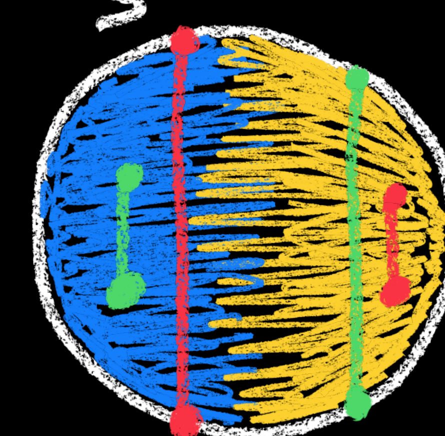


clasp singularities



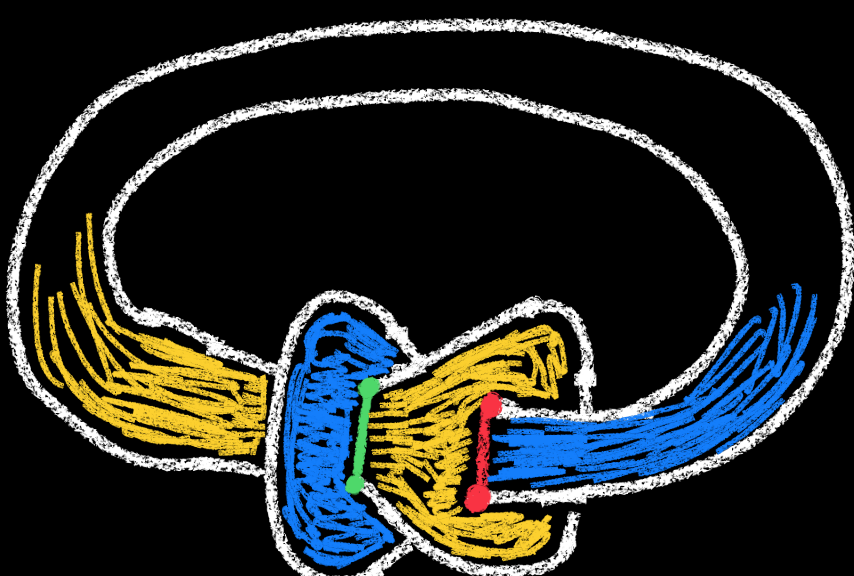


ribbon singularities

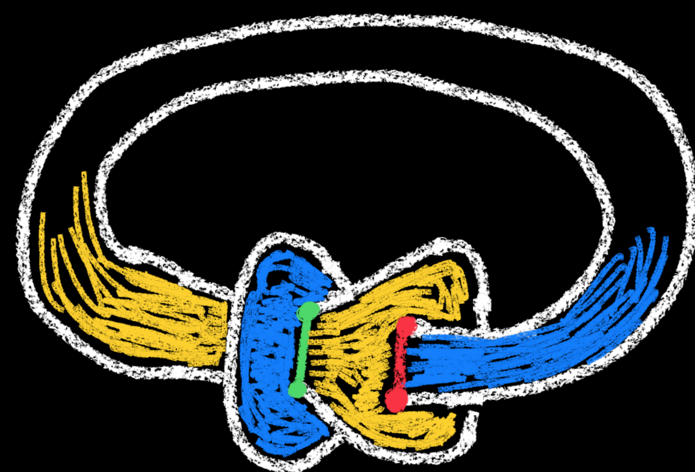


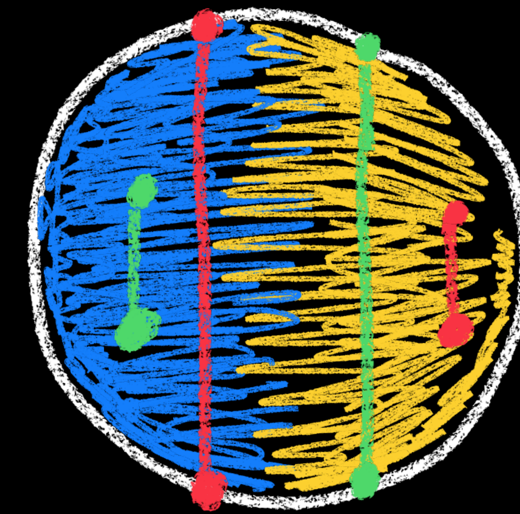
__FACT:__ the only $K \subseteq S^3$ which bounds an *embedded* disk in S^3 is the __unknot__.

__DEF__ A knot $\subseteq S^3$ bounding a disk with only ribbon singularities is a __RIBBON knot__.

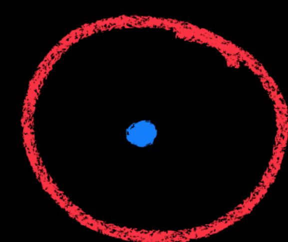


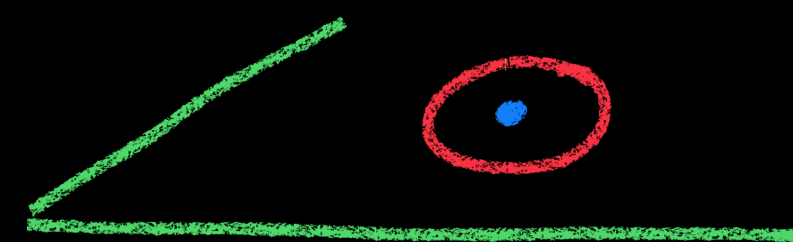
Example:

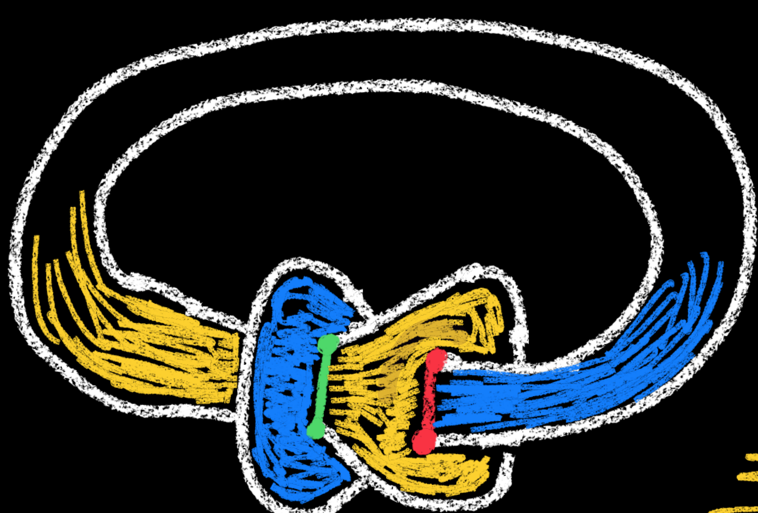




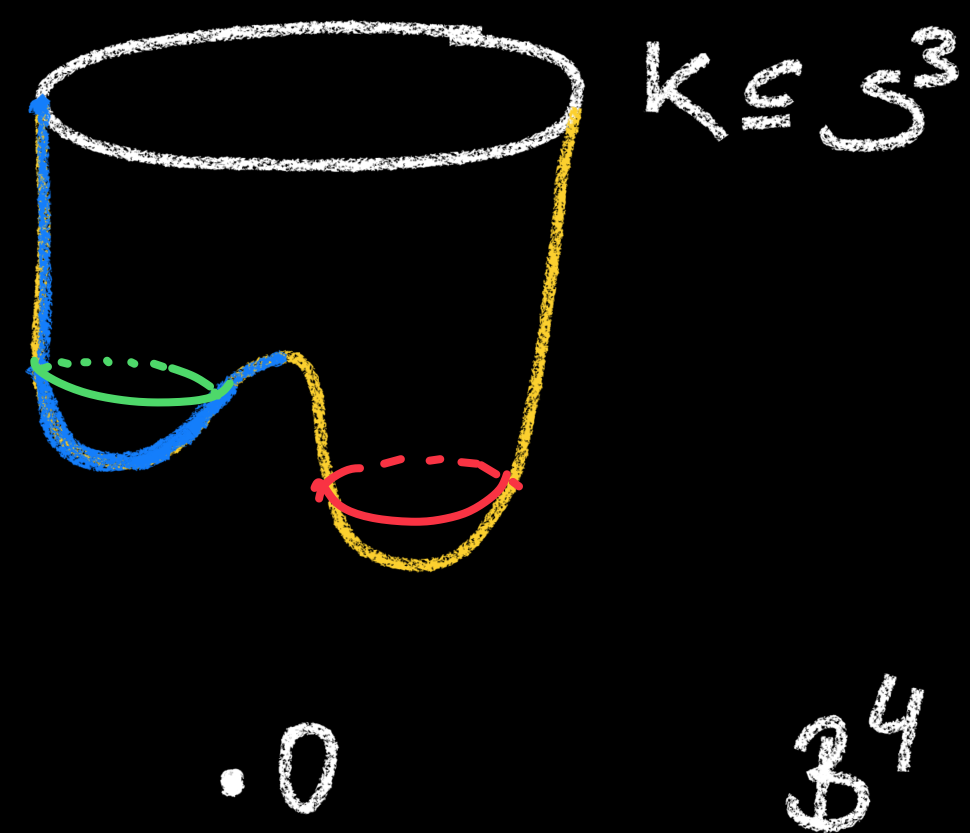
warm up.



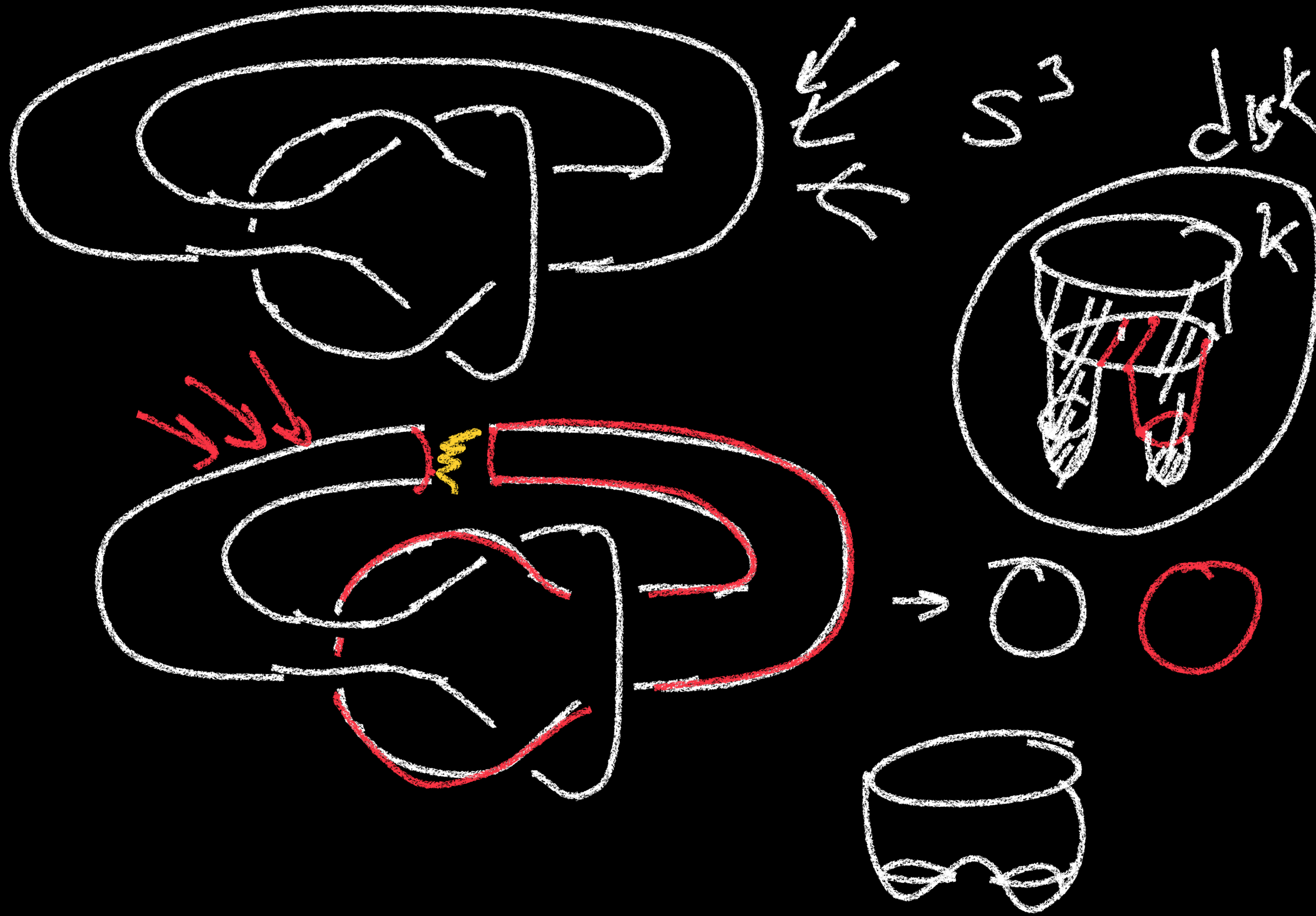




$\rightsquigarrow$

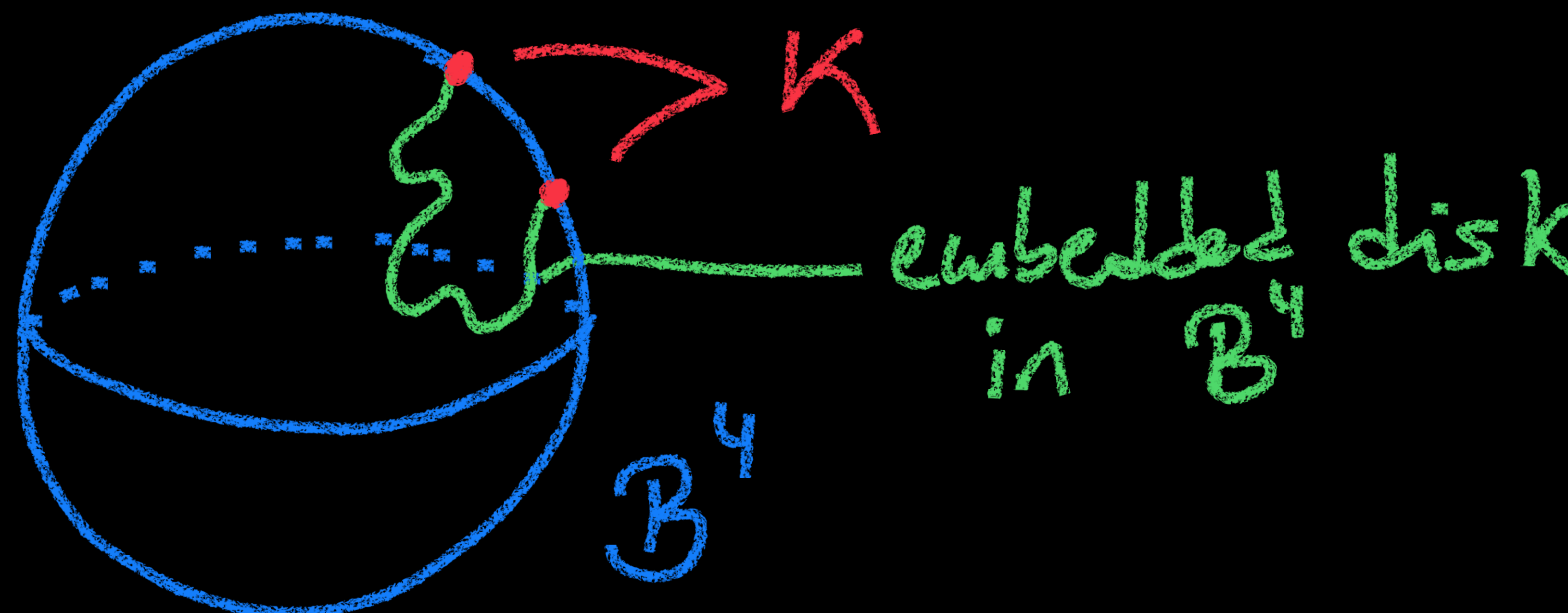


A DIFFERENT DESCRIPTION : MOVIES



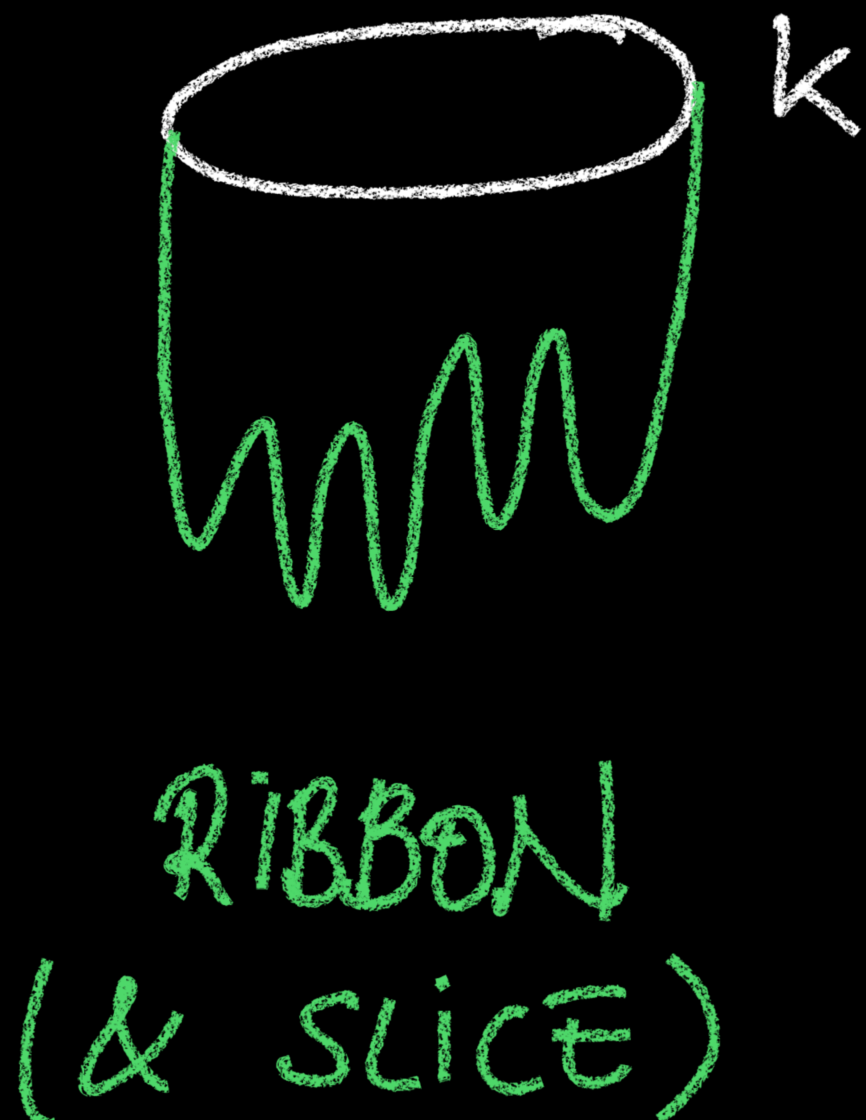
Q: Since $S^3 = \partial B^4$, which knots bound embedded disks in B^4?

cartoon:



DEF A knot $\subseteq S^3$ is SLICE if it bounds a smoothly embedded disk in B^4.

We have shown: ribbon knots are slice and they bound disks $\subseteq B^4$ with <u>NO</u> maxima for the radial function in B^4.



BIG QUESTION: the slice ribbon conj.

Fox ('62): slice $\Rightarrow$ ribbon.

- Key word: Knot concordance
  (arXiv: >250 papers on last 10 years)

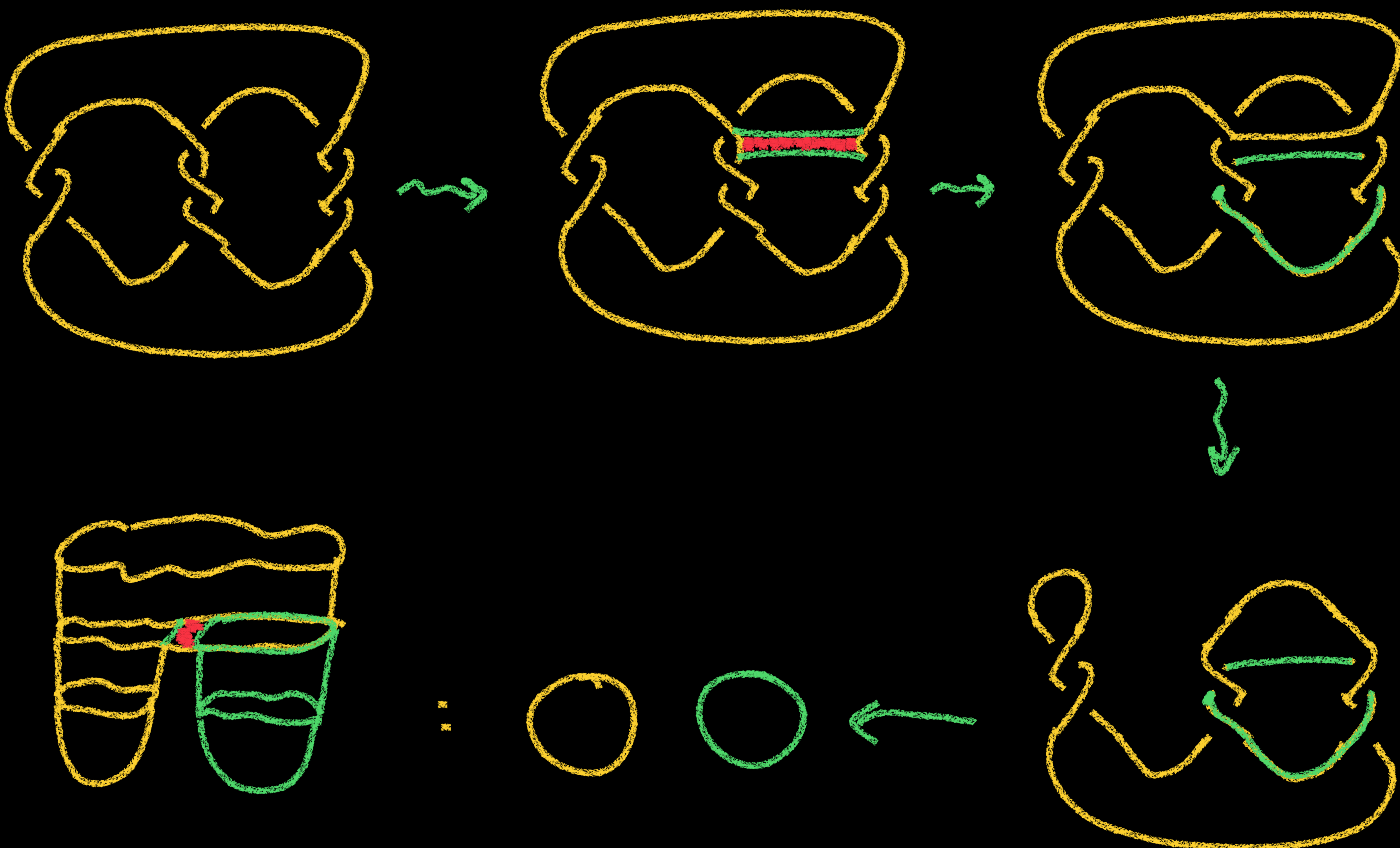
<u>HOW TO APPROACH IT?</u>

1. Find a counterexample.
2. Show that every slice disk is ribbon.

"1.5" Show 2 for <u>families</u> of knots.

$P(p,q,r) :=$ 

SLICE PRETZEL KNOTS: P(p, q, −q)



HOW TO OBSTRUCT SLICENESS?

if K is slice then...

$\sigma(K) \in \mathbb{Z}$ vanishes

$\tau(K) \in \mathbb{Z}$ khov. hom vanishes

$\Delta_K(t) = f(t) \cdot f(t^{-1})$

$Y^3_K = \partial(\mathbb{Q}HB^4)$

etc.

Piccirillo (Annal's 2020) Conway knot is not slice

WHAT WE KNOW TODAY ABOUT PRETZELS

$P(p,q,r) = K$ p,q,r odd.

K slice $\Leftrightarrow$ $q=-r$

r even, p,q odd.

If $\underline{p=-q} \Leftarrow$ $\underline{K \text{ slice}}$

$$P\left(a,-a-2,-\frac{(a+1)^2}{2}\right) \quad a \equiv 1, 11, 37, 47 \; [60]$$