Interacting Particle systems and (S)PDEs:
Modelling, Analysis and Computation

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Overview

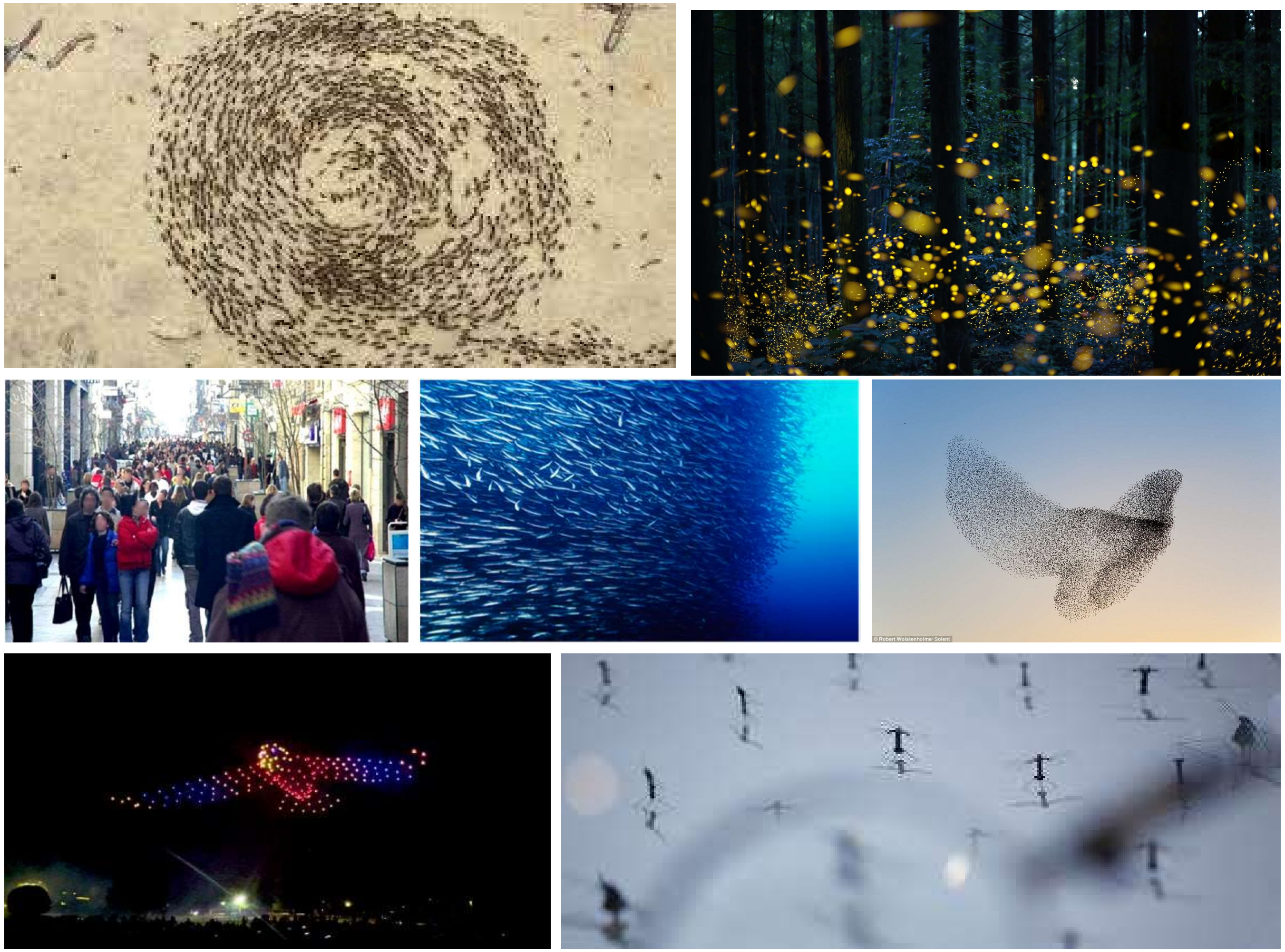
- Something about me
- Overview of collective behaviour
    - Interacting Particle System (IPS) and Collective Behaviour
    - Importance of the field
    - Why am I interested in this?

- Interacting Particle Systems and PDEs
- Interacting Particle Systems and Stochastic PDEs – open problems
- Modelling issues

Something about me

- Short CV:
  - PhD 2012 at Imperial College
  - 1 Year postdoc at Warwick
  - 1 year Chapman Fellowship at Imperial
  - Lecturer then Associate Professor at HW

- What have I done since/what are my roles?
  - Built my research group, composed of 3 PhD students, 2 postdocs
  - Awarded Whittaker Prize of the Edinburgh Mathematical Society (2019)
  - Member of the European Mathematical Society
  - Director of Postgraduate Studies for MACS
  - Co-Organiser of the One-World Dynamics seminar
  - Sat various national panels to award grants

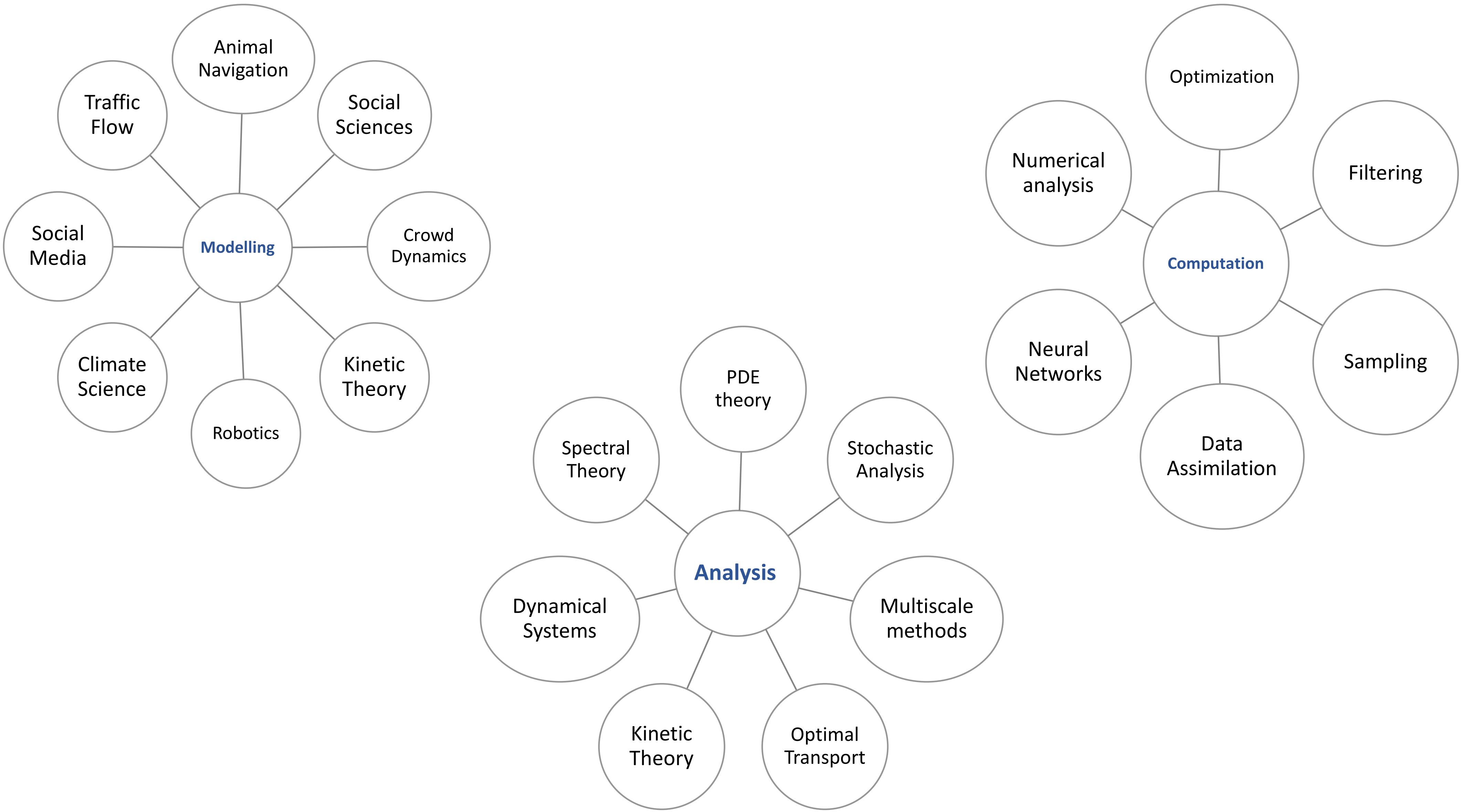


Interacting Particle Systems and Collective behaviour

Huge range of authors, from a number of disciplines: Degond, Carrillo, Parisi, Couzin, Barre', Flandoli, Wolfram, Gomes, Evans, Sherratt, Painter, Eftimie, Reich, Stuart, Spiliopoulos, Bertozzi, Bertini, Meleard, Pulvirenti, Calvez, MacKay, Kolokoltsov, Perthame,Zatorska, Theil...

`We feel that the study of human collective behavior must become the *crisis discipline* response to changes in our social dynamics.'

Stewardship of global collective behaviour
PNAS, June '21



Modelling
Animal Navigation
Traffic Flow
Social Sciences
Social Media
Crowd Dynamics
Climate Science
Robotics
Kinetic Theory
Analysis
PDE theory
Spectral Theory
Stochastic Analysis
Dynamical Systems
Multiscale methods
Kinetic Theory
Optimal Transport
Computation
Optimization
Numerical analysis
Filtering
Neural Networks
Sampling
Data Assimilation

Why am I interested in this?

Can we make long-term predictions?

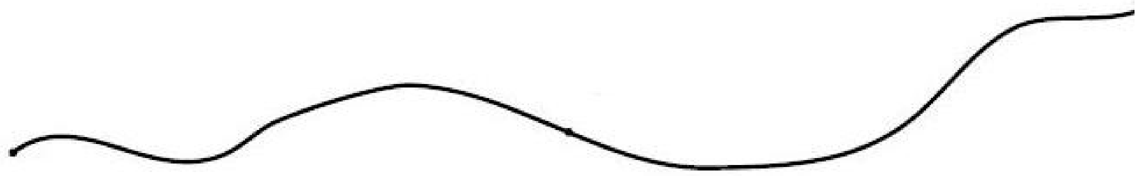
- Uniform in time approximations
- Processes with multiple equilibria

Processes with multiple equilibria

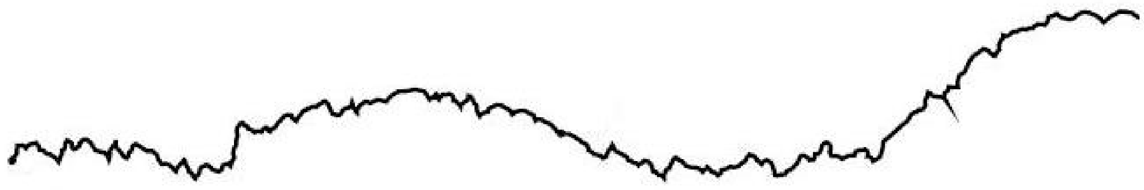
Relate initial datum to equilibrium selected by the dynamics

What is noise? ODEs, S(O)DEs, PDEs, SPDEs

ODE $dx(t) = f\big(x(t)\big)dt \quad x(t) \in R$



S(O)DE = ODE + noise $dx(t) = f\big(x(t)\big)dt + dB_t$



Similarly for PDEs and SPDEs

Interacting Particle systems and PDEs

Interacting Particle System

$$dX_t^i = \frac{1}{N}\sum_{i=1}^{n} K\left(X_t^i - X_t^j\right) dt + dB_t^i$$

i=1, …, N

$$\Downarrow \quad N \to \infty$$

Partial Differential equation

$$\partial_t \rho_t = \partial_x (K * \rho_t)\, \rho_t + \partial_x^2 \rho_t$$

Interacting Particle systems and SPDEs

Interacting particle system with common noise

$$dX_t^i = \frac{1}{N}\sum_{i=1}^{n} K\left(X_t^i - X_t^j\right) dt + dB_t^i + dW_t$$

$$\Downarrow$$

Stochastic Partial Differential Equation

$$\partial_t \rho_t = \partial_x (K * \rho_t)\rho_t + \partial_x^2 \rho_t + \partial_x \rho_t \, dW_t$$

- The above convergence is crucial in filtering/data assimilation

- How do we obtain this?

$$\partial_t \rho_t = \partial_x (K * \rho_t)\rho_t + \partial_x^2 \rho_t + dW_t$$

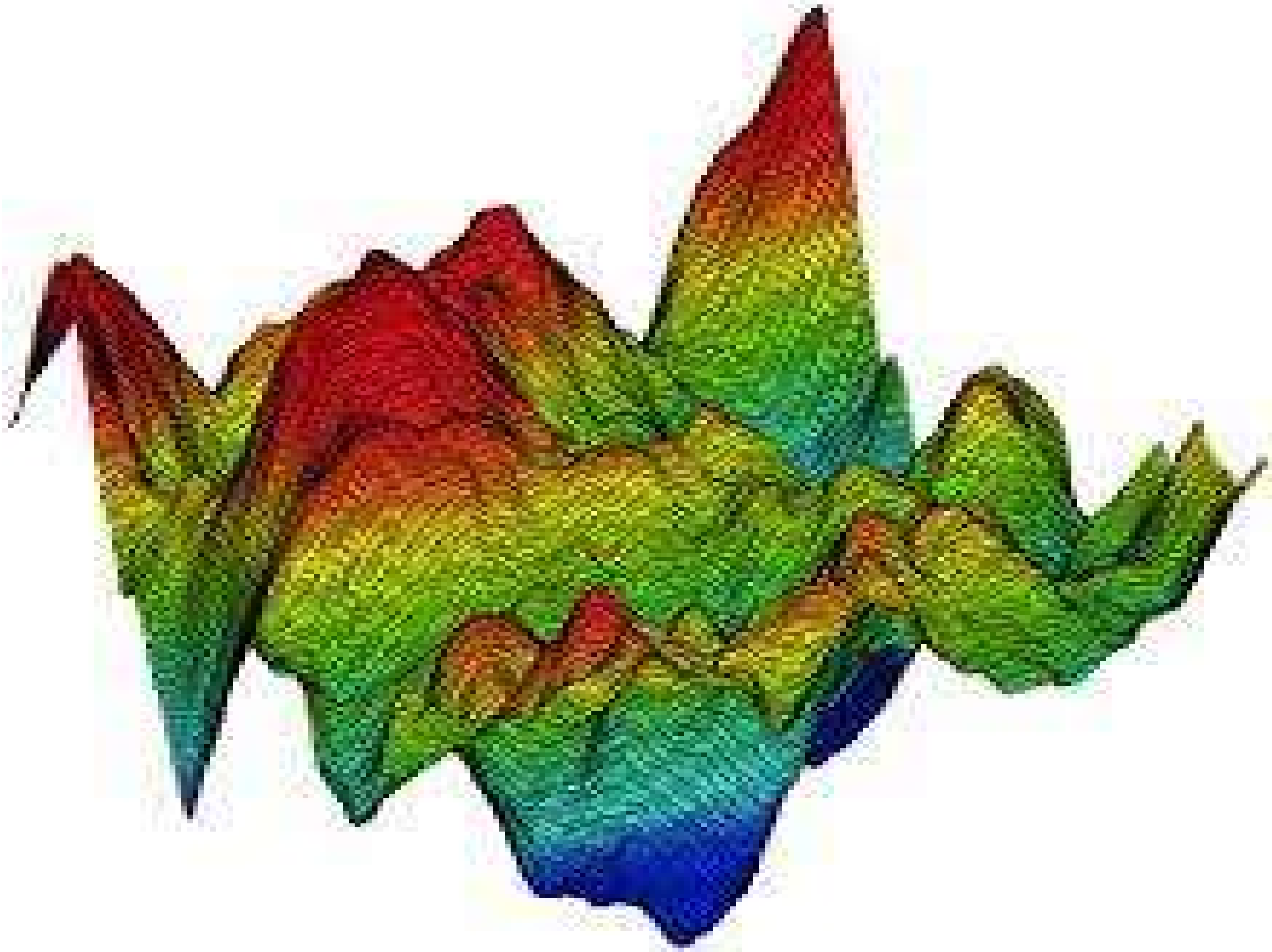
Modelling Issues – interaction rules

- Who interacts with whom?
  - `All- to-all' interaction (mean field)
  - Sparse interactions
  - Anisotropic interactions

- Type of interaction
  - Velocity alignment
  - Aggregation (position)
  - Short range repulsion/long range attraction
  - Leader-follower models
  - Particles with `intrinsic features'

- Properties of the particle system vs properties of the limiting (S)PDE
- Is it always appropriate to let $N \to \infty$?

Dual questions biology - simulation



Happy to take any questions!