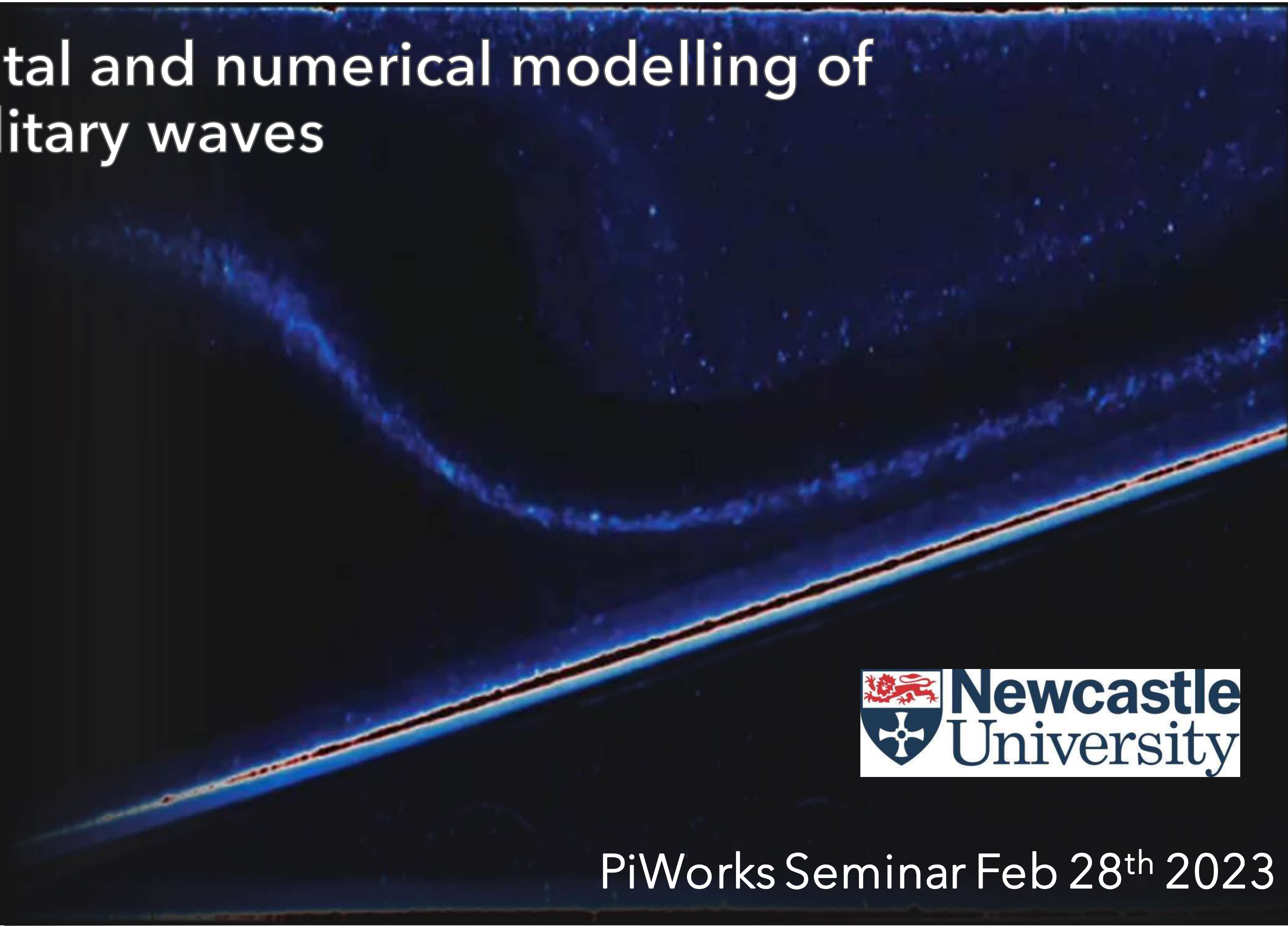
Experimental and numerical modelling of internal solitary waves

Magda Carr



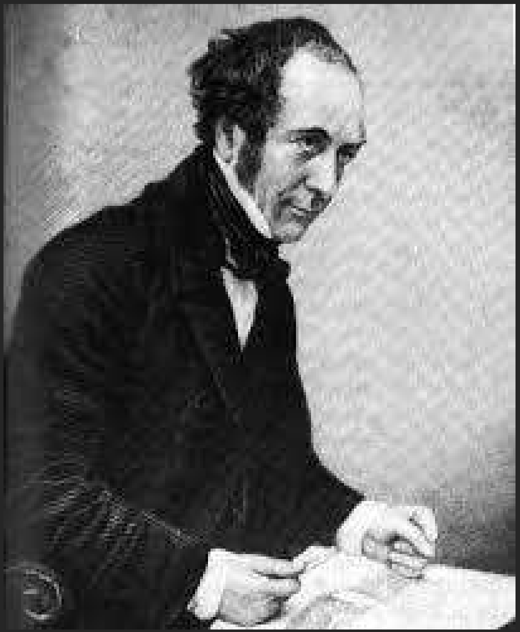
Newcastle University

PiWorks Seminar Feb 28th 2023

Overview

- Background & Motivation
- Experimental Method
- Numerical Method
- Some Examples
- A little bit about me & my career

Solitary Waves



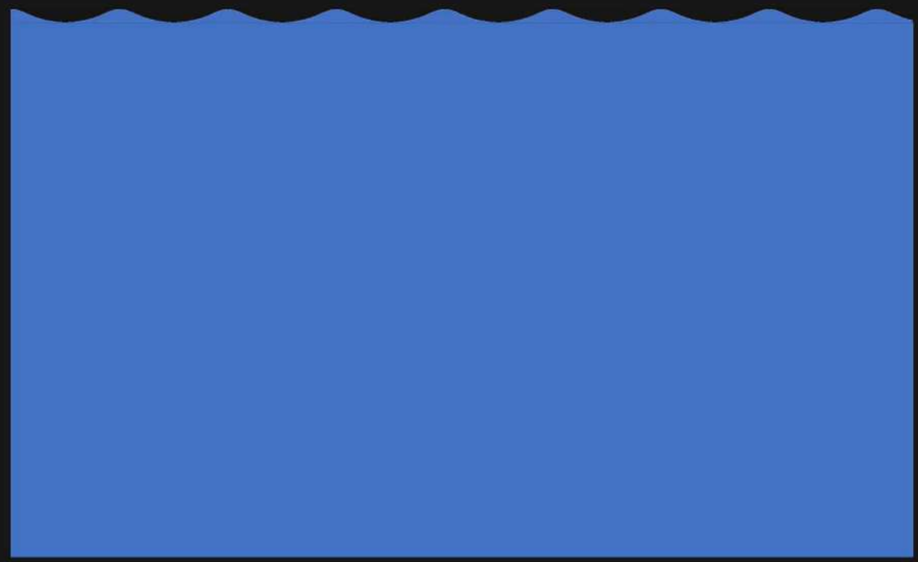
John Scott Russell (1834)

Scott Russell Aqueduct

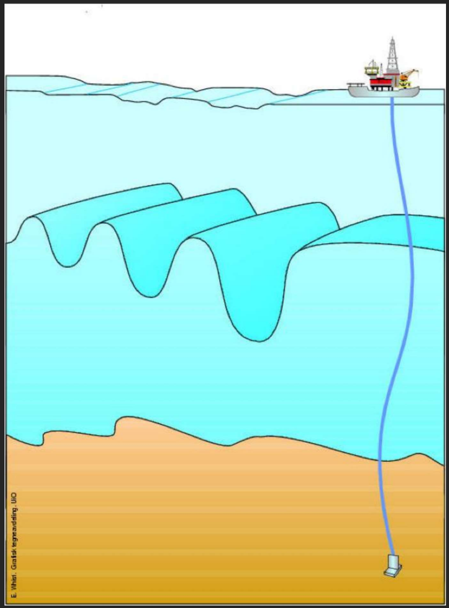
Scott Russell Aqueduct

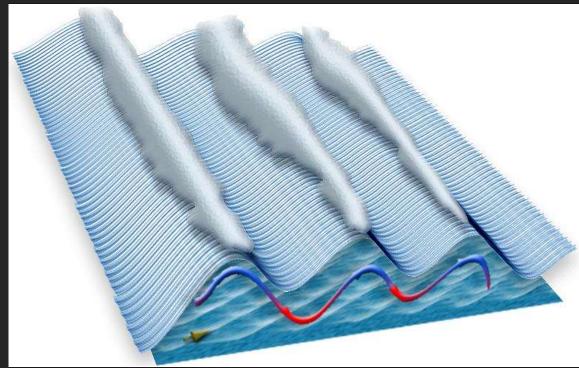
- “Wave of translation” - travel large distances without change of form
- Speed depends on amplitude - larger/faster
- Do not merge - solitary behaviour

Internal Solitary Waves



Internal Solitary Waves





- Travel on density interfaces within stably stratified fluids; balance between nonlinear wave steepening and linear wave dispersion
- They occur in many geophysical settings including coastal zones & river outflows, estuaries, lakes, reservoirs & fjords, oceans & marginal seas and the atmosphere.
- Gaseous plasmas, liquid crystals, acoustics, solid state physics, optical fibres, Bose-Einstein condensates, etc.

Helfrich & Melville (2006), Lamb (2014), Boegman & Stastna (2019), *Annual Review of Fluid Mechanics.*

Properties of ISWs

- Wave speeds 0.1-1 m/s; front 10-100 km; length 0.1-1 km, amplitudes 10-100m.
- Rank-ordered wave packet
- a=120 m, H=340 m: Duda *et al.* 2004 (S China Sea)
- a=240 m, c=2.55 m/s: Huang *et al.* 2016 (S China Sea)

Forth Road Bridge (155m)

Physical Oceanography

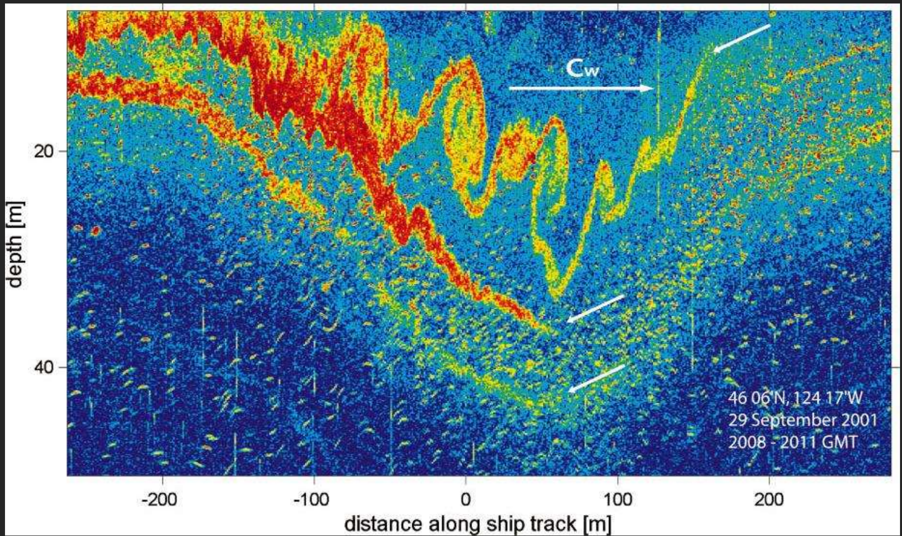


Fig 1. Moum *et al.* 2003. Oregon continental-shelf.

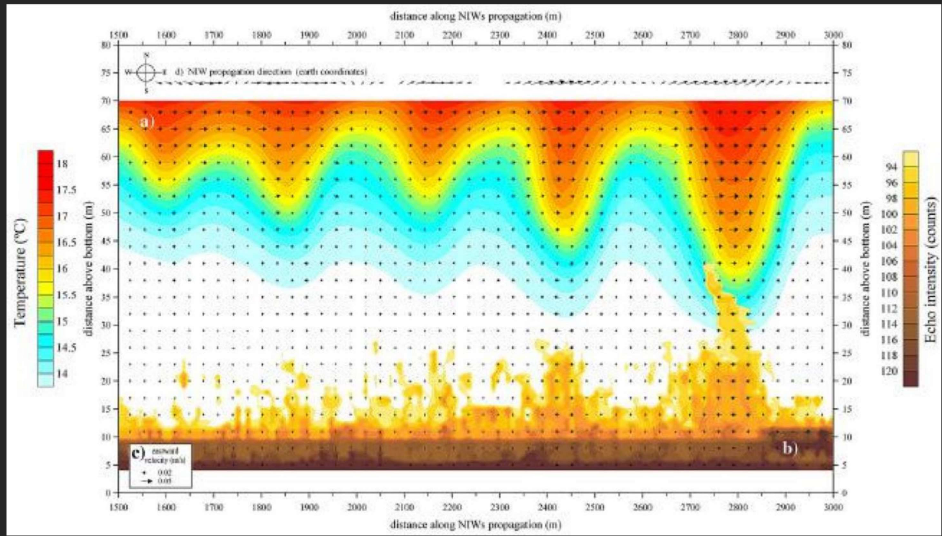
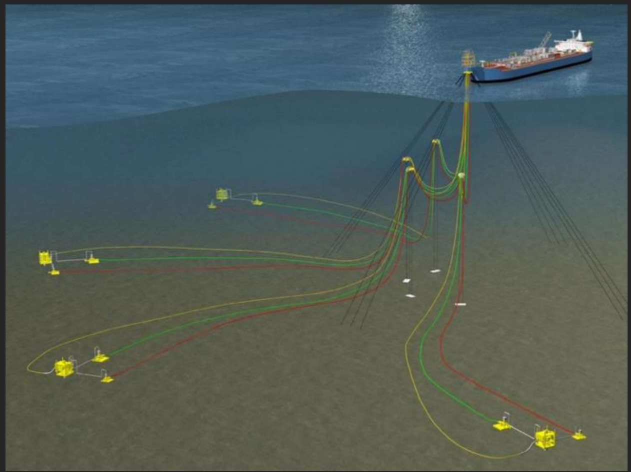
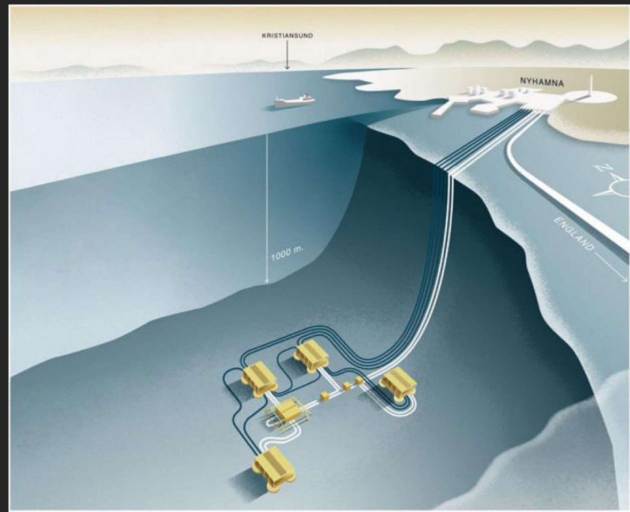


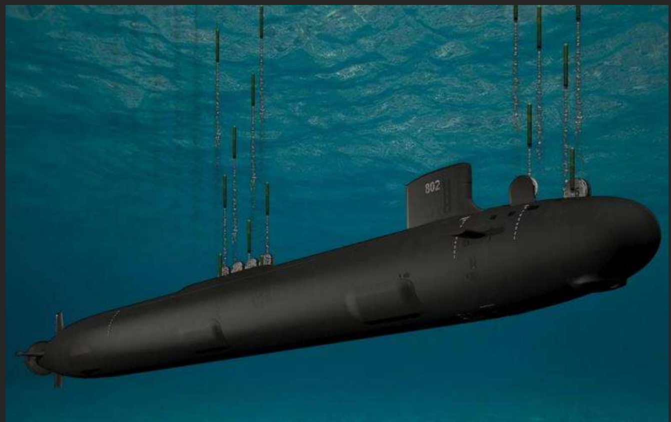
Fig 2. Quaresma *et al.* 2007. Western Portuguese mid-shelf.

- Source of momentum & mixing
- Vertical transport of heat & nutrients
- Resuspension of sedimentary material
- Transport of mass over large distances.

Applications







Remote Sensing of ISWs

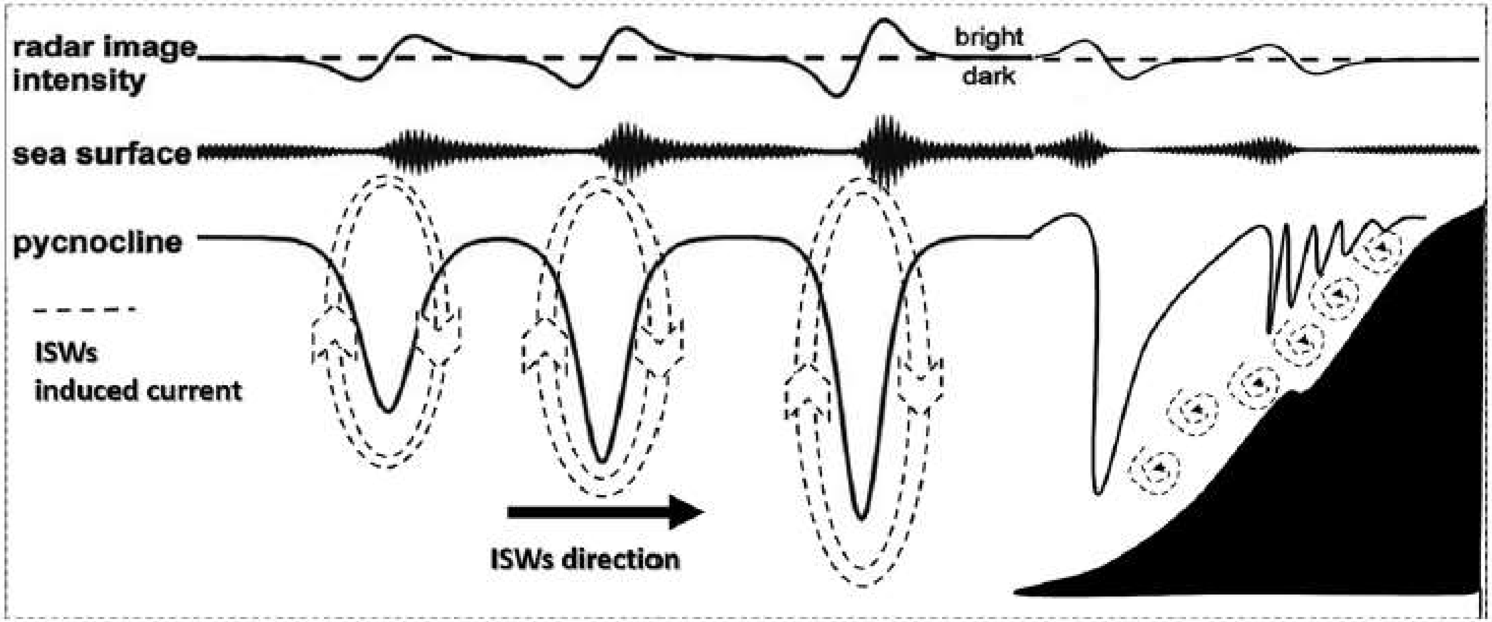
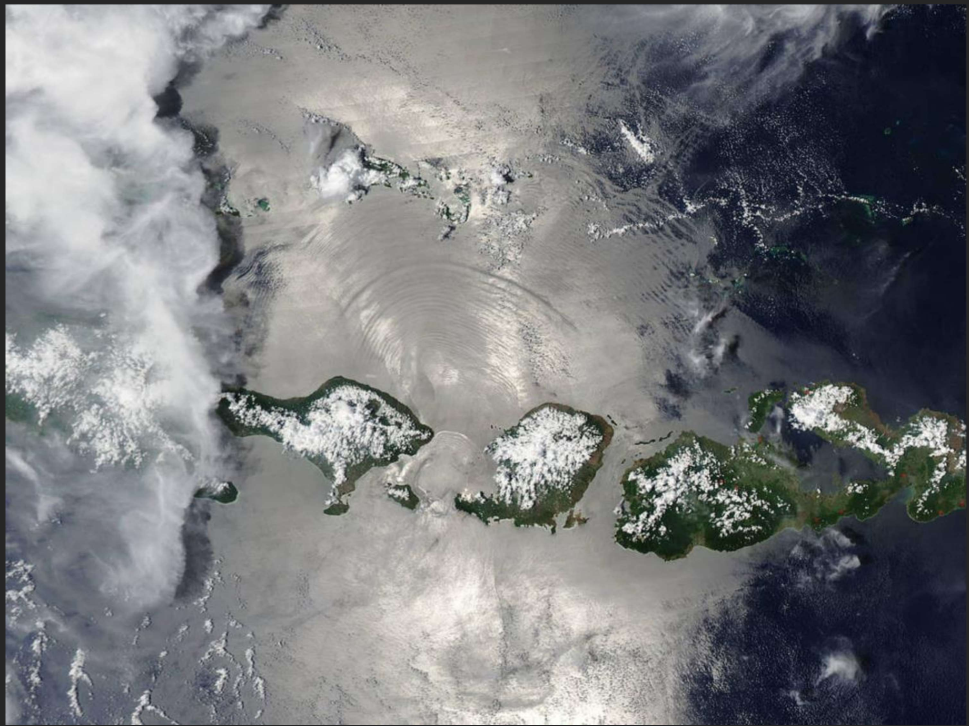


Figure 2. Schematic plot of remote sensing mechanism of internal solitary waves [adapted from earth.esa.int].

Shen *et al.* 2020. JGR Oceans.

Remote Sensing of ISWs



NASA satellite photos of (i) the Lombok strait, (ii) Hainan Island and (iii) the strait of Gibraltar.

Global Distribution of ISWs

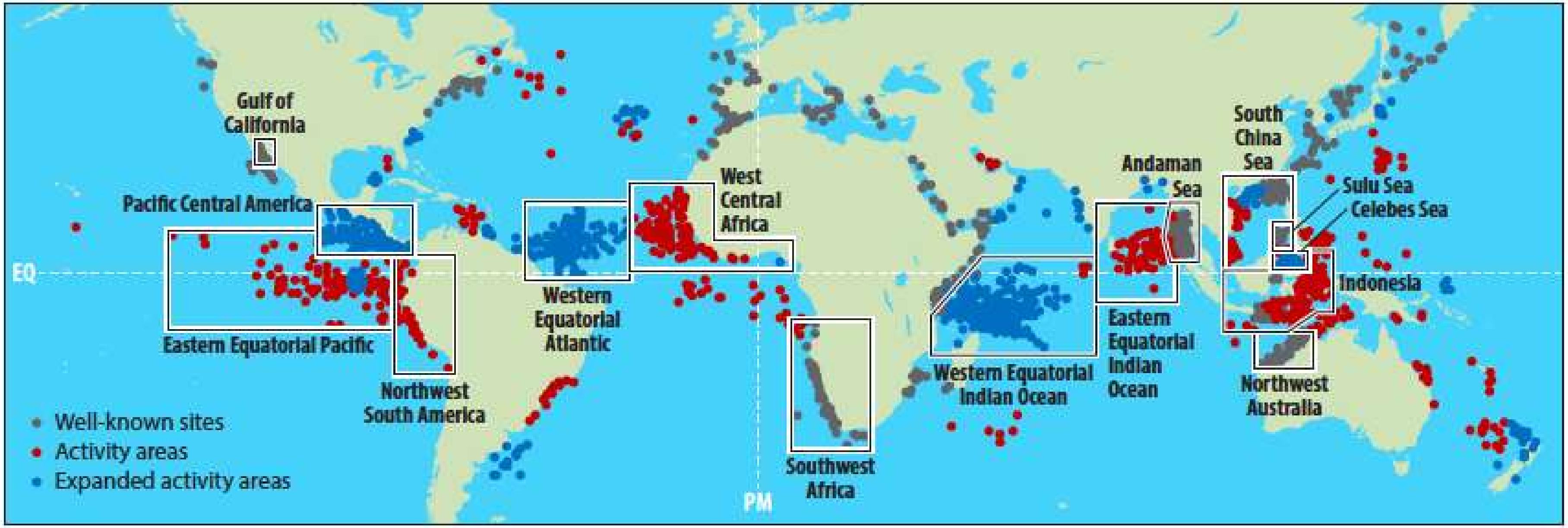


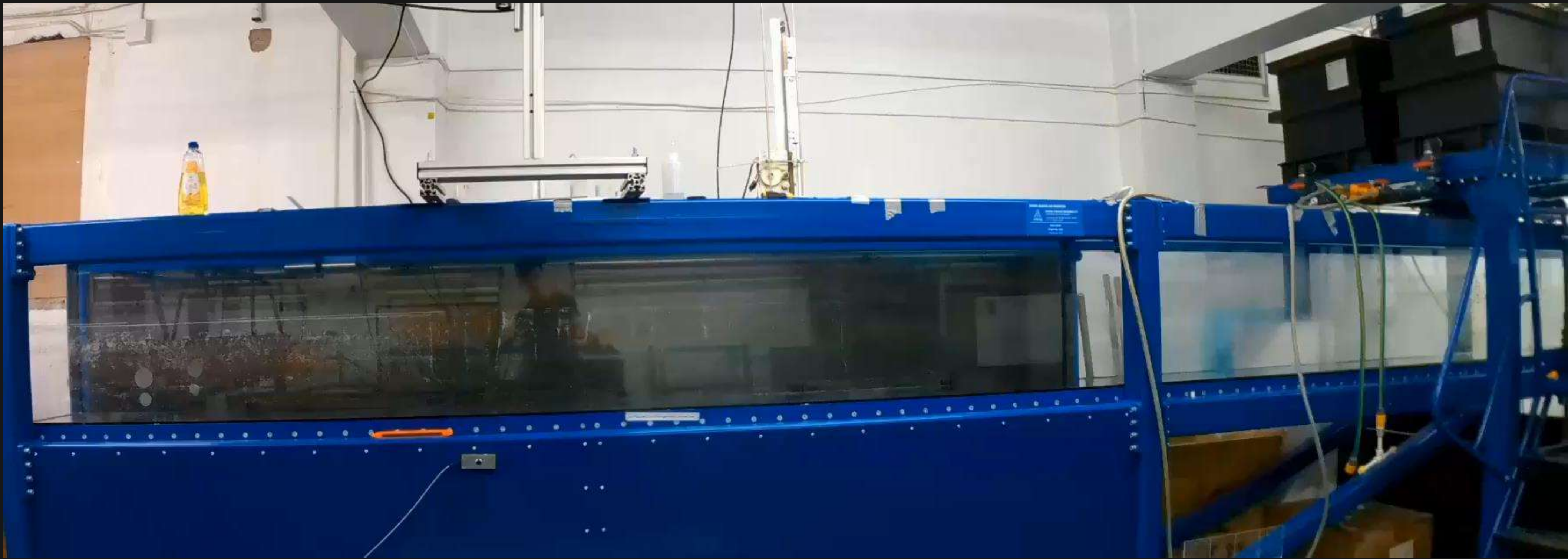
Figure 1

Global distribution of internal solitary waves. Internal waves observed from August 2002 through May 2004. Shown are well-known occurrence sites (*gray*), new areas of activity (*red*), and areas of geographically expanded activity (*blue*). Figure adapted from Jackson (2007), with permission from Wiley.

- Jackson (2007) observed ISW packets on nearly every coastline worldwide on 95% of observation days. Image taken from Boegman & Stastna (2019), *Annual Review of Fluid Mechanics.*

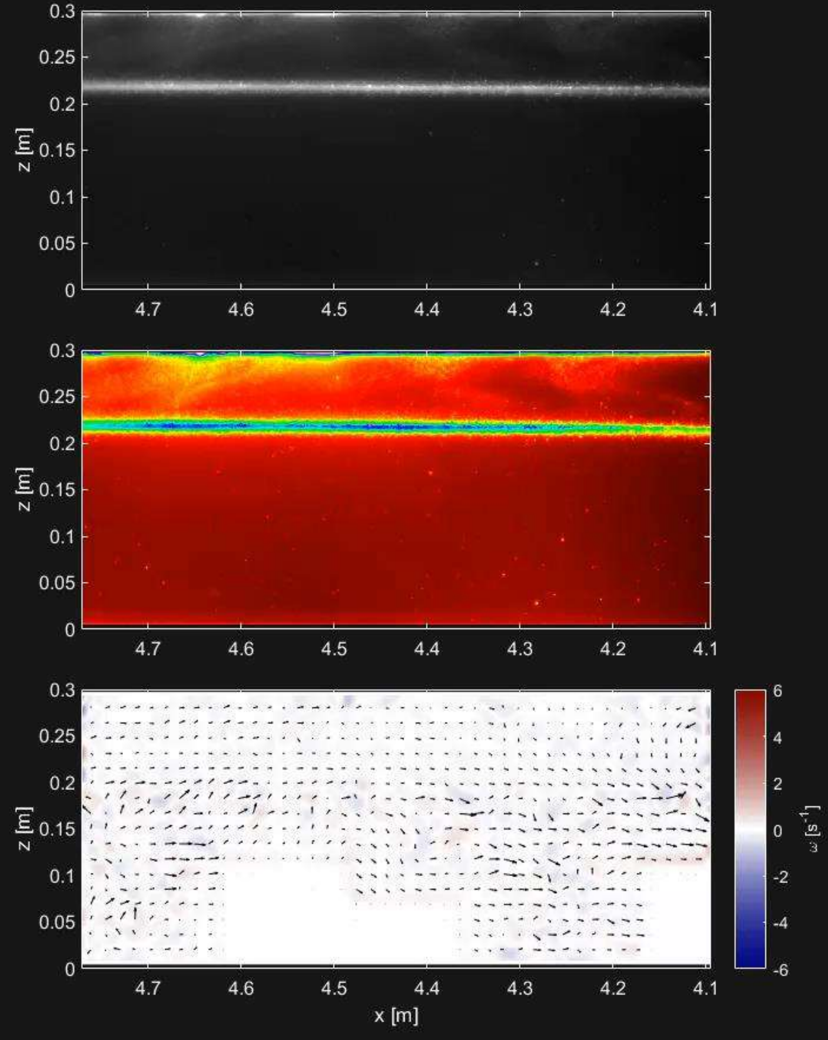
How do we model Internal Solitary Waves ?

Experimental Methods



Experimental Methods

- Particle Image Velocimetry (PIV) – synoptic velocity field.
- Microconductivity Sensors – traverse of the density field.



Numerical Model – Spectral Parallel Incompressible Navier-Stokes Solver (SPINS)

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla}\, \vec{u} = -\frac{1}{\rho_0}\, \vec{\nabla} P + \upsilon \nabla^2 \vec{u}\; - \frac{\rho g}{\rho_0}\, \hat{k}$$

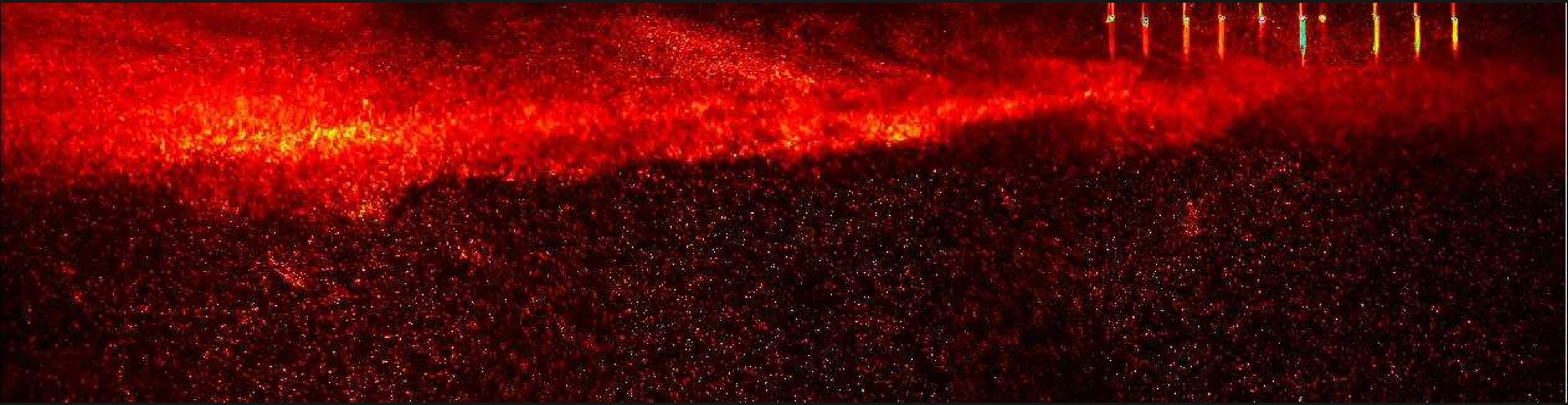
$$\nabla \cdot \vec{u} = 0$$

$$\frac{\partial \rho}{\partial t} + \vec{u} \cdot \vec{\nabla} \rho = \kappa \nabla^2 \rho$$

Subich *et al.*, 2013; *Int. J. Numer. Meth. Fluids*, **73**, 103-129

Some Results

Laboratory Observation of an unstable ISW



- Fructus, Carr, Grue, Jensen & Davies. *J. Fluid Mech.* (2009).
- Carr, Franklin, King, Davies, Grue & Dritschel. *J. Fluid Mech.* (2017).

Numerical Results

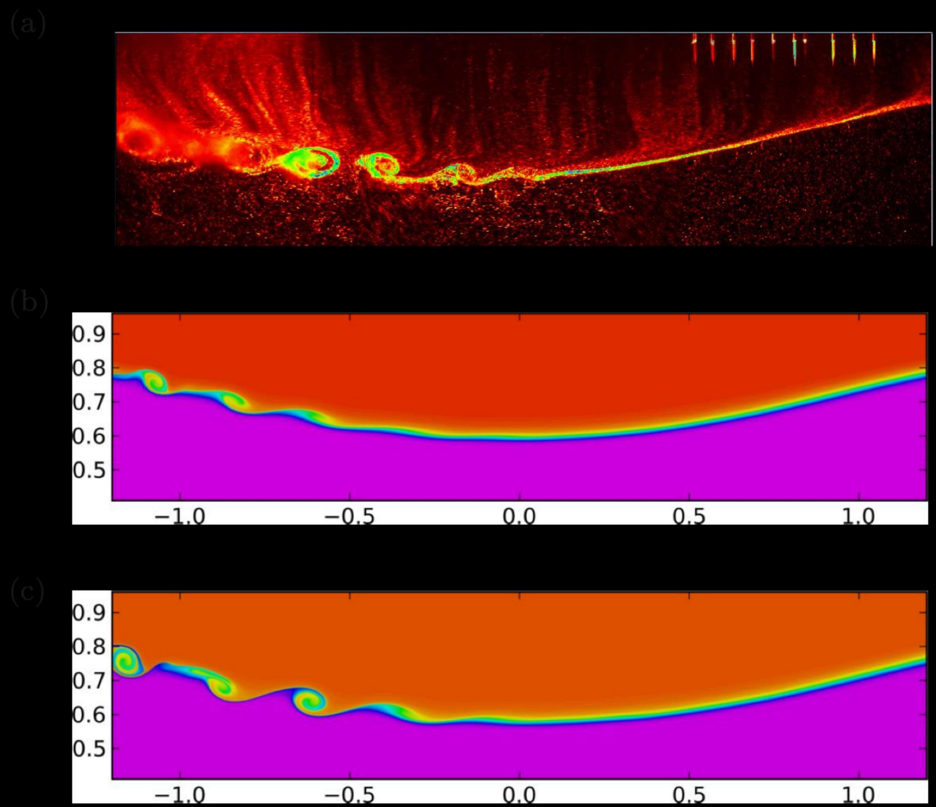
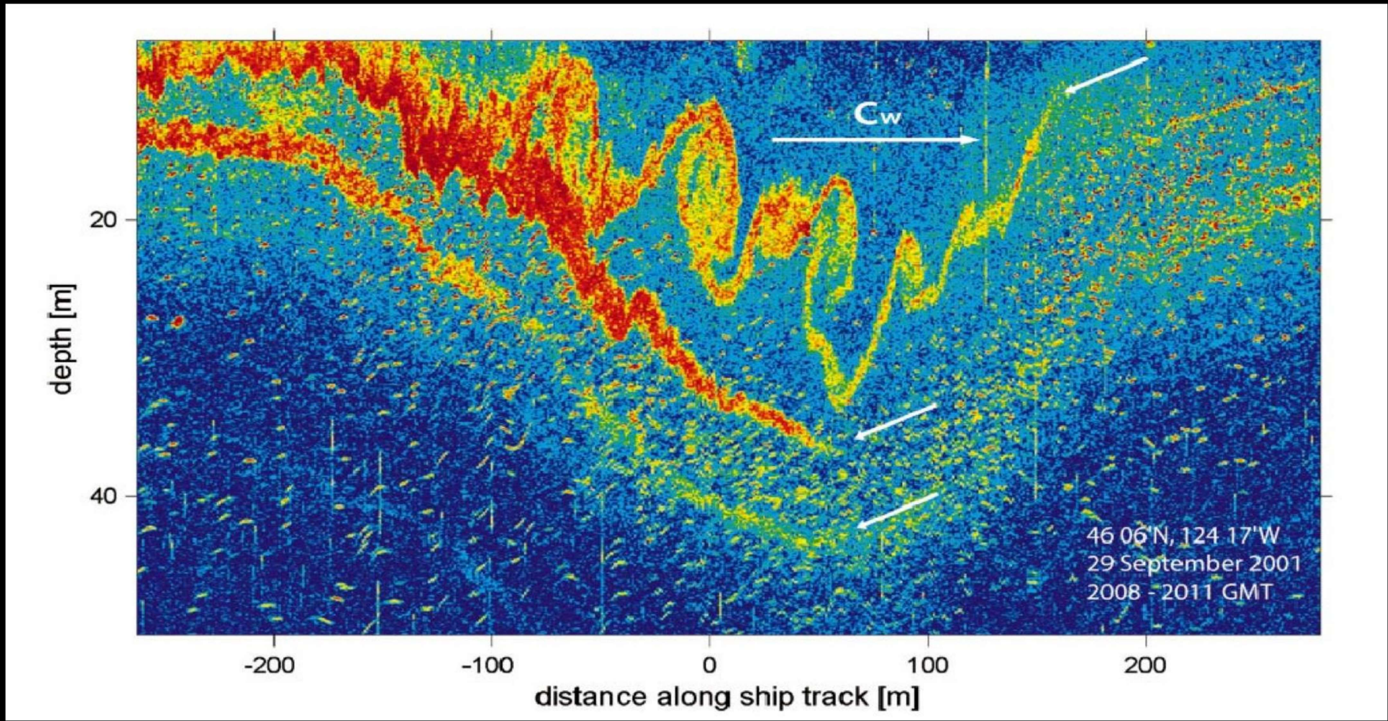


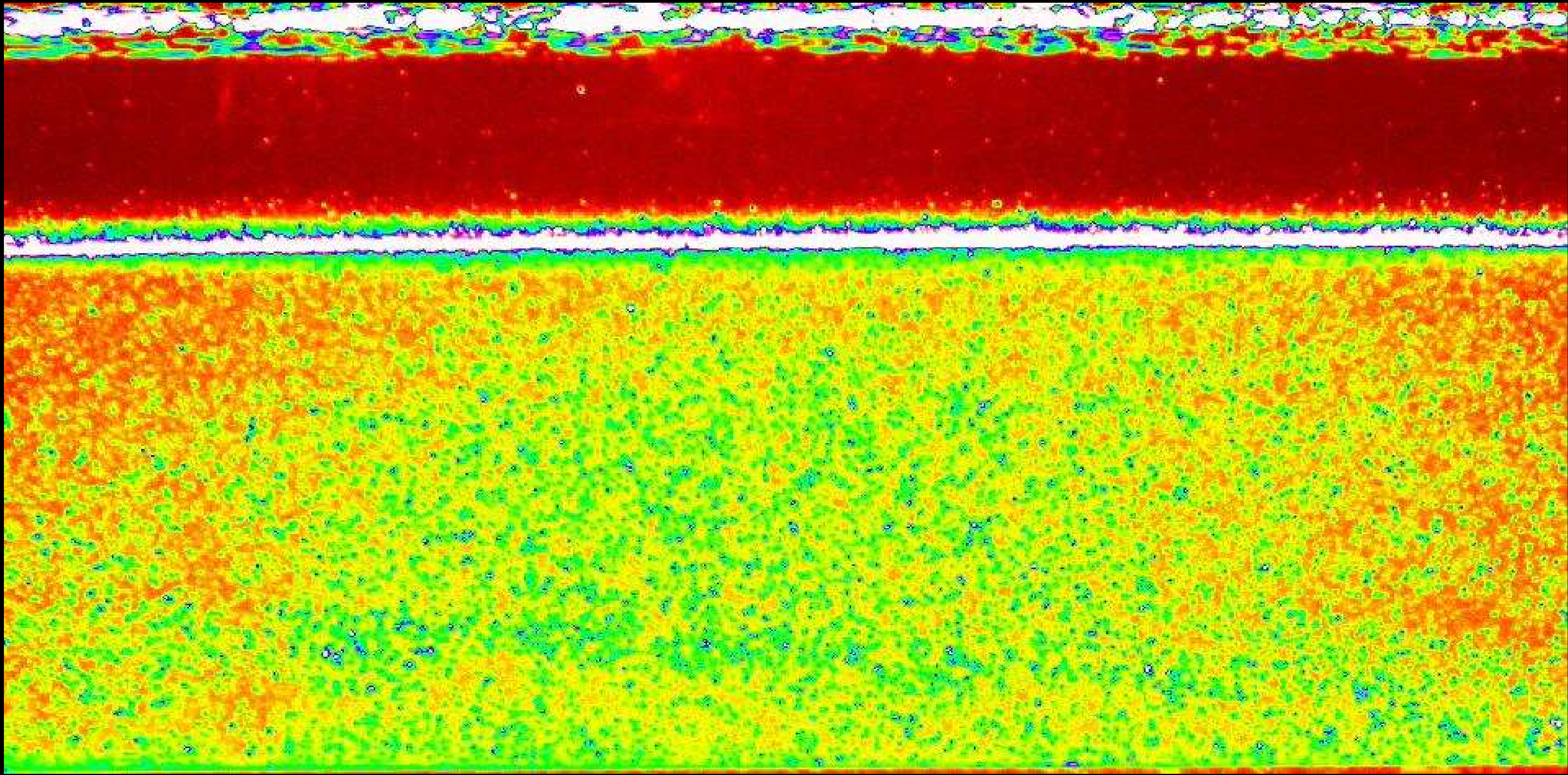
Fig: (a) Experimental image and (b) corresponding numerically computed buoyancy field when a_lower and (c) a_upper are matched with the laboratory wave depicted in (a).

Field Observation

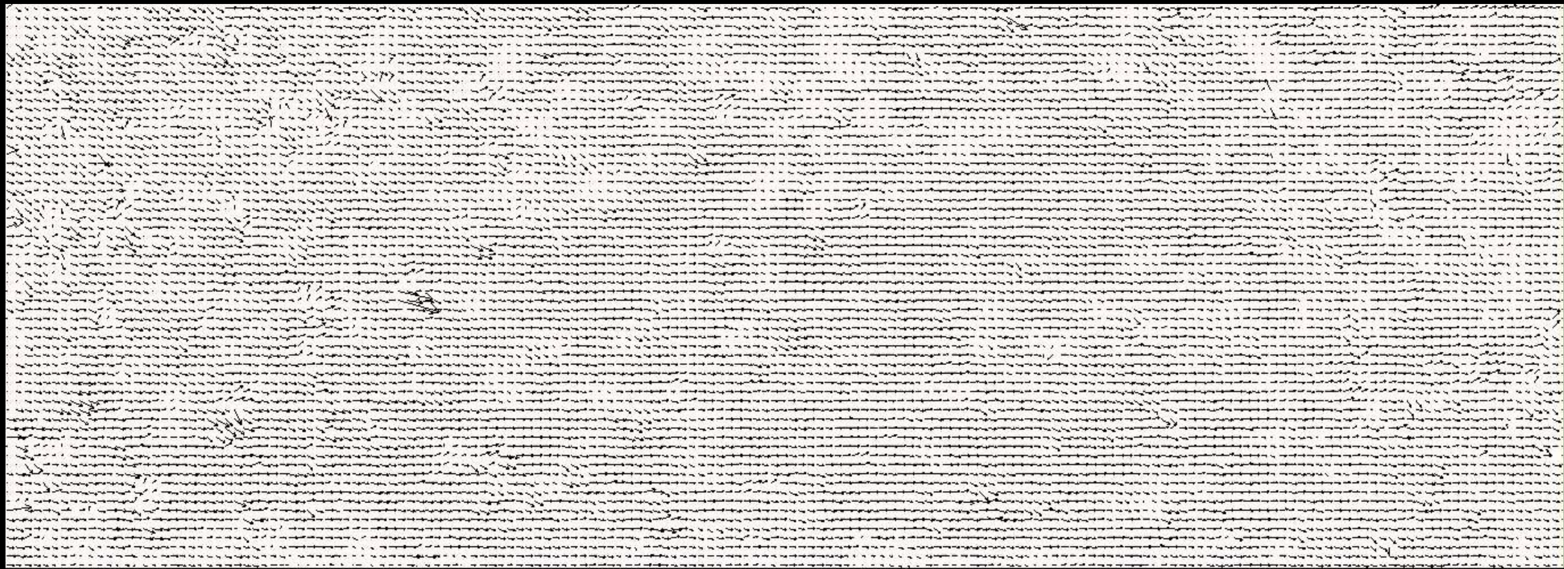


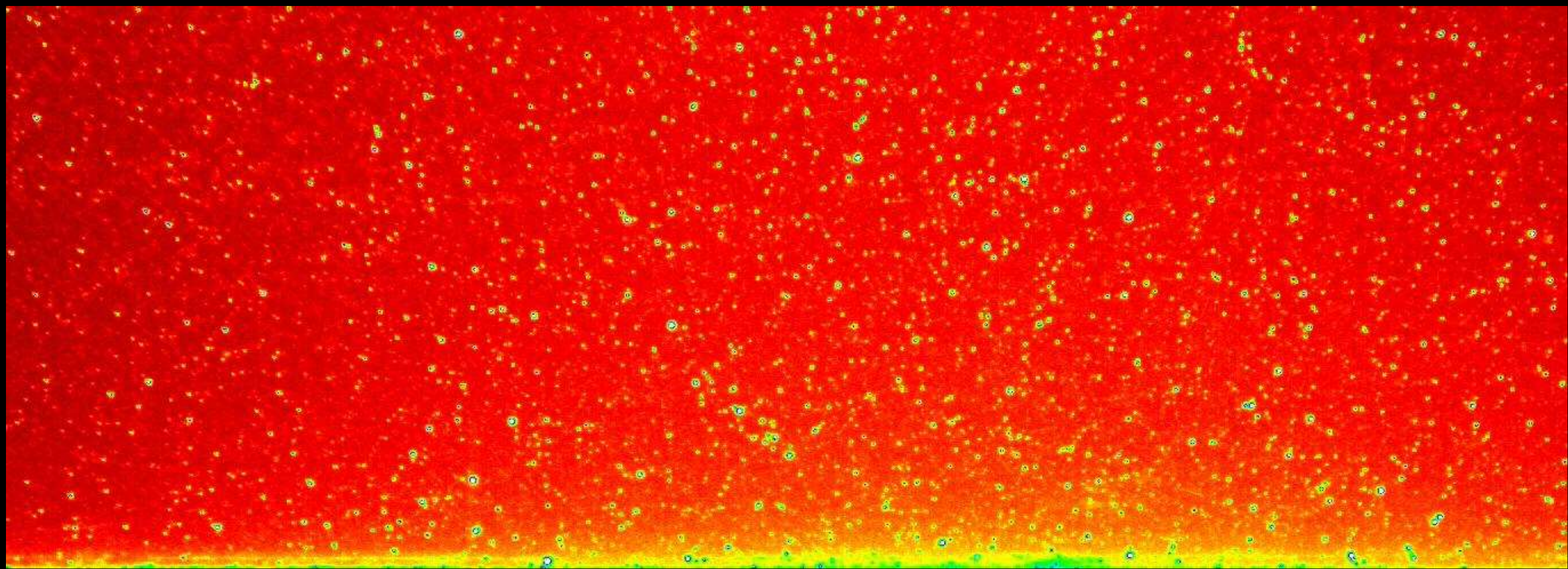
Moum *et al*. 2003. Oregon Continental Shelf.

Carr & Davies. *Phys. Fluids.* (2006)

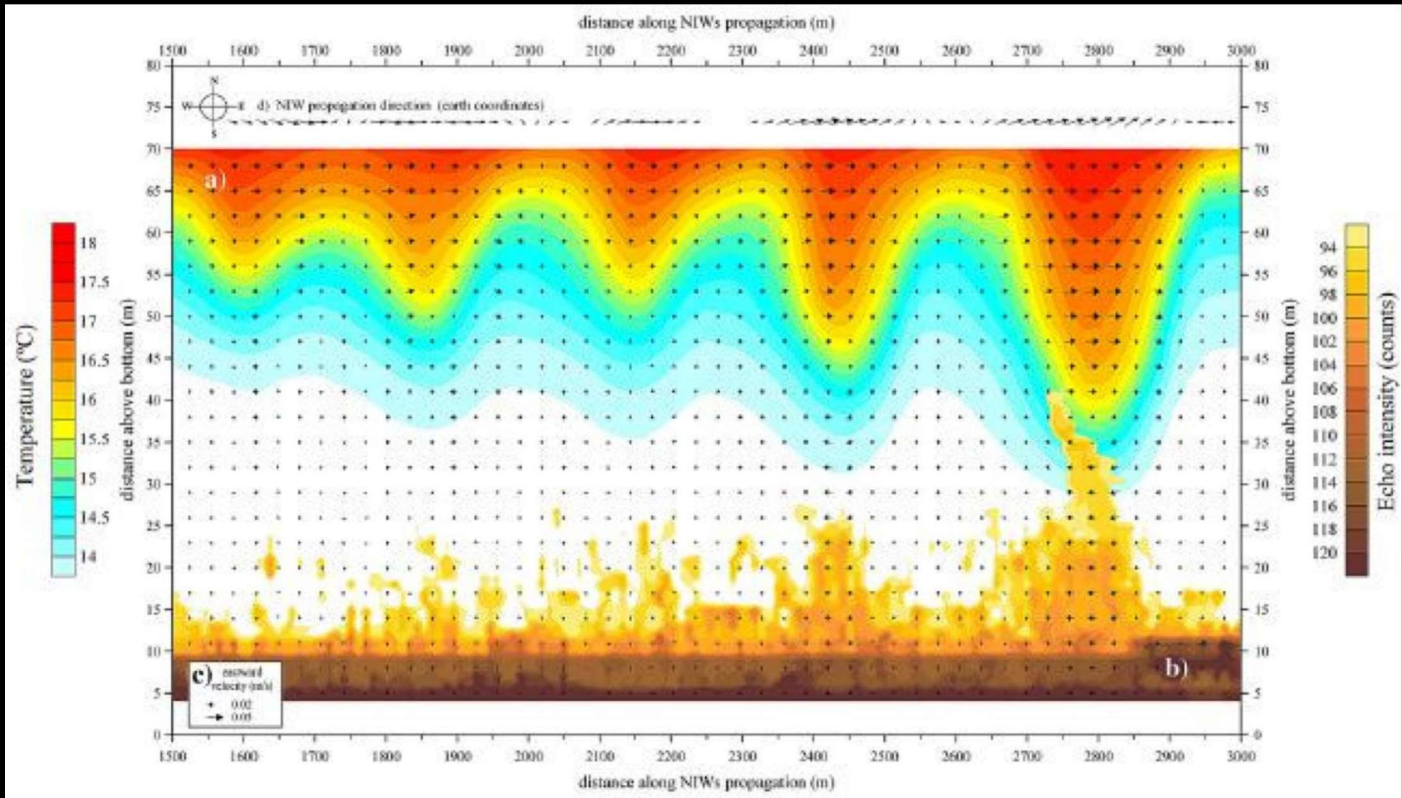


Carr, Davies & Shivaram. *Phys. Fluids.* (2008)





Field Observation



Quaresma *et al*. 2007. Western Portuguese Mid-shelf.

Classification of shoaling breakers

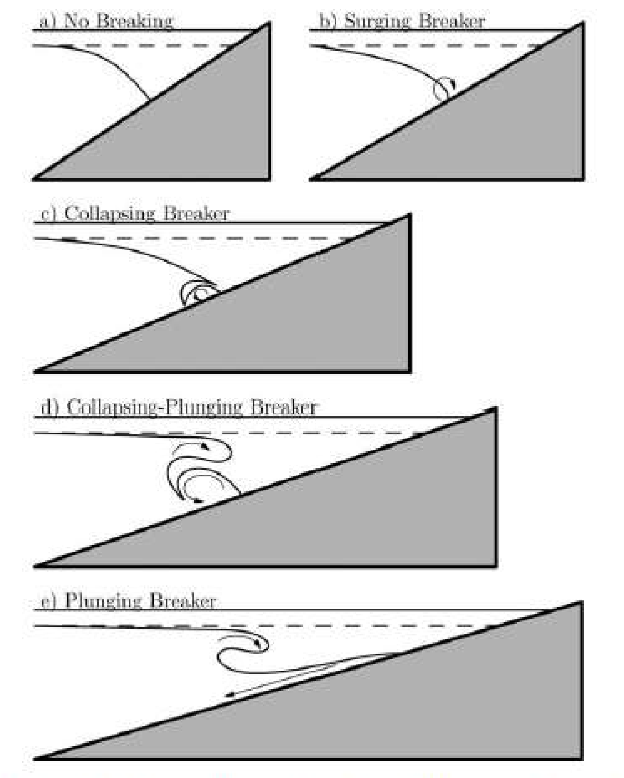


Figure 1. Schematics illustrating different breaking mechanisms for solitary waves of depression approaching a constant slope. Sketches are representative of the flow around the time of maximum breaking depth.

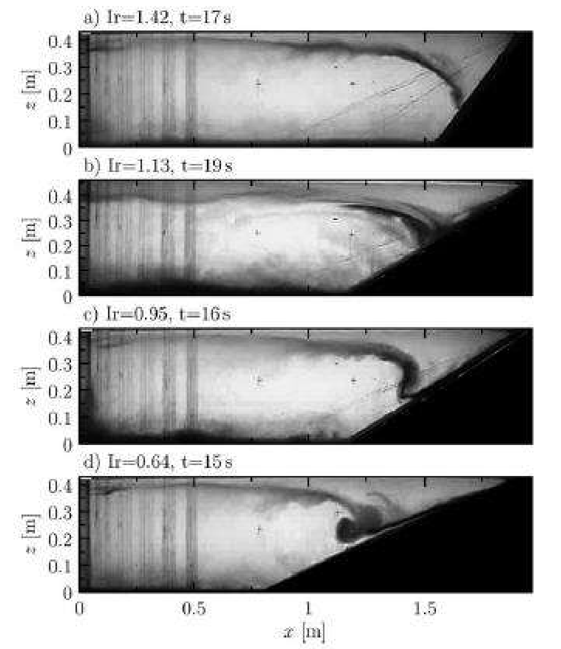


Figure 4. Snapshots from four experiments in the short tank distinguishing different breaking mechanisms according to the indicated Iribarren number: (a) surging breaker (vertical interface at boundary), (b) collapsing breaker (boundary-layer separation), (c) collapsing-plunging breaker (boundary-layer separation and overturning aloft), and (d) plunging breaker (overturning aloft). Note movies of each of these experiments can be viewed as supporting information to this paper.

- Sutherland *et al.* 2013. JGR.

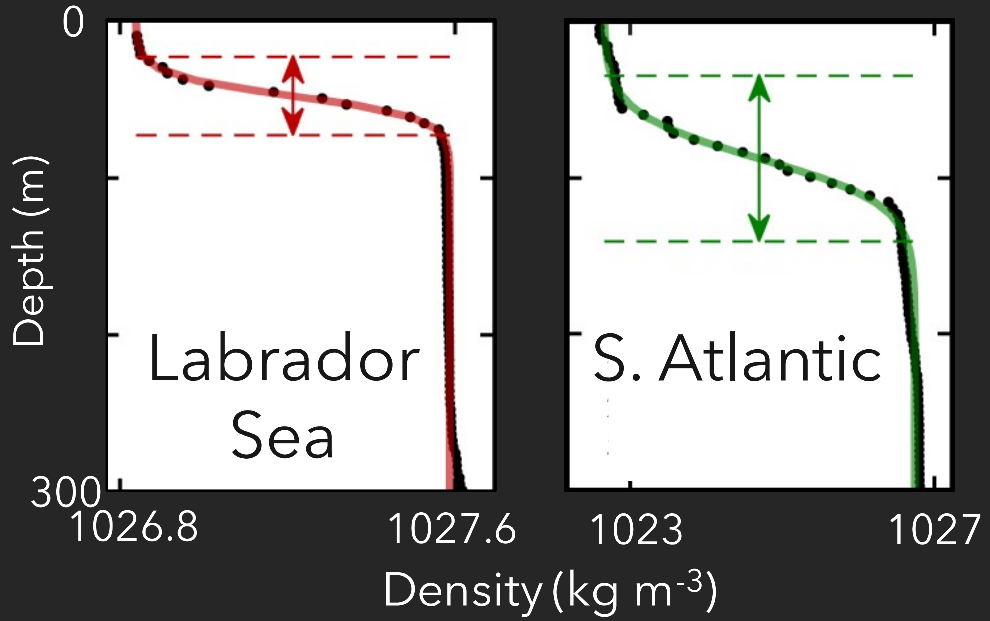
Classification of shoaling breakers

$$Ir = \frac{s}{\sqrt{a/\lambda}}$$

Iribarren Number

- Qualitative change from plunging, to collapsing to surging as *Ir* increases (Boegman *et al.* 2005)
- On gentle slopes fission followed by bolus formation (Aghsaee *et al.* 2010)

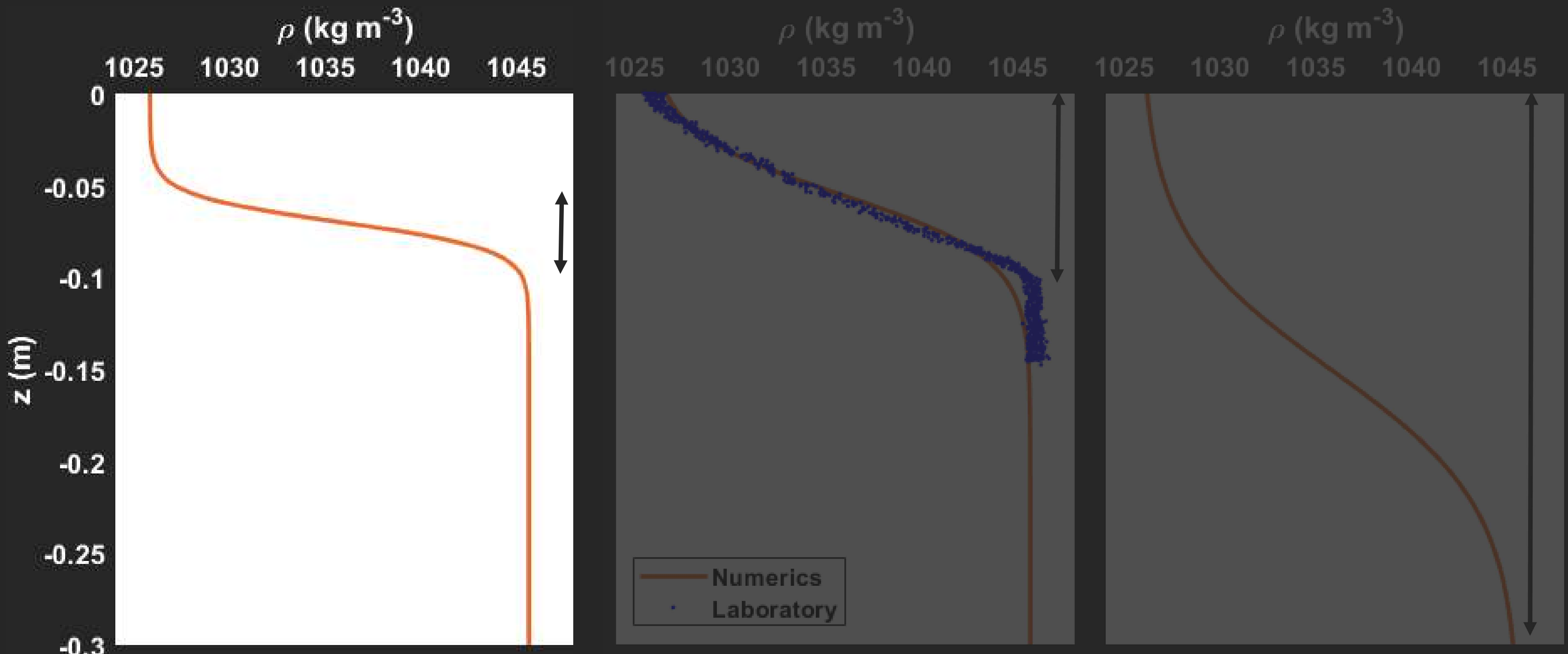
Stratification in the Ocean



$$\rho(z) = \rho_2 + \Delta\rho \tanh\left(\frac{z - z_{pyc}}{h_{pyc}}\right)$$

Vieira and Allshouse, 2020; *JFM*.

Stratification Investigated

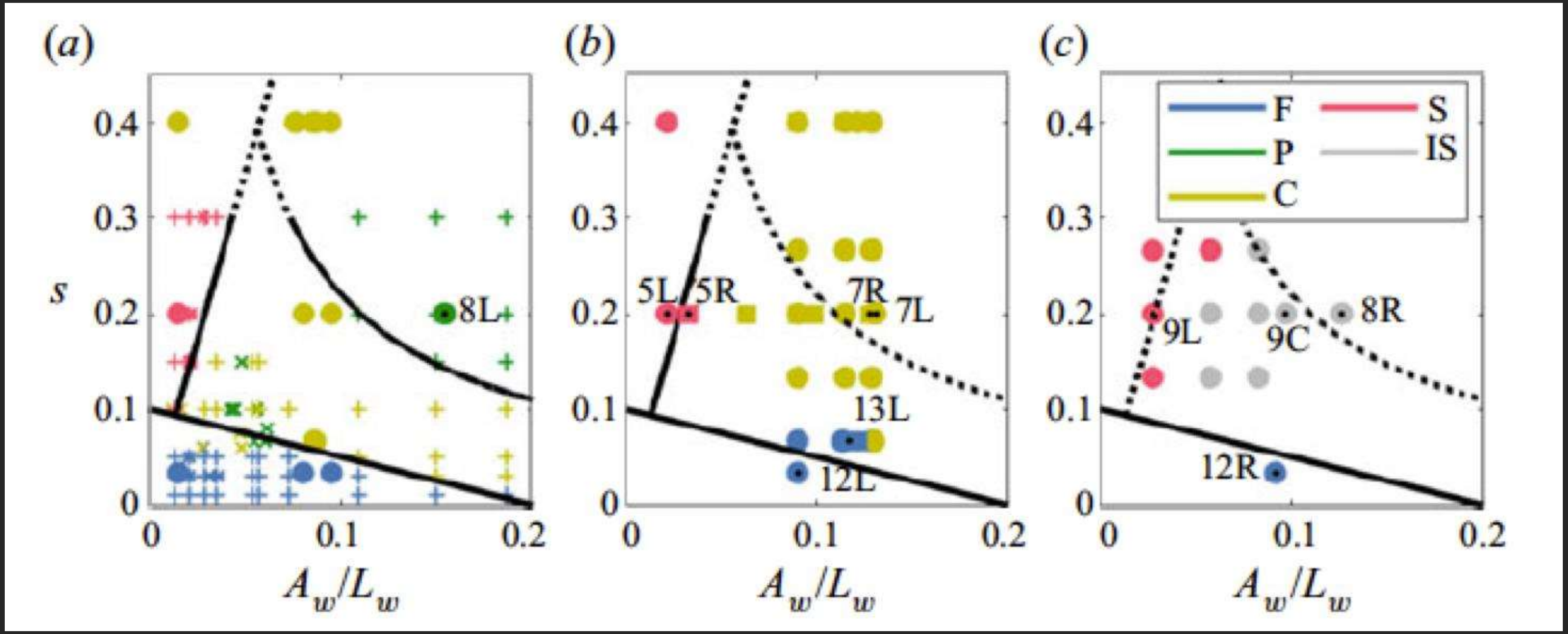


Thin Pyc.

Surface Stratification

Broad Stratification

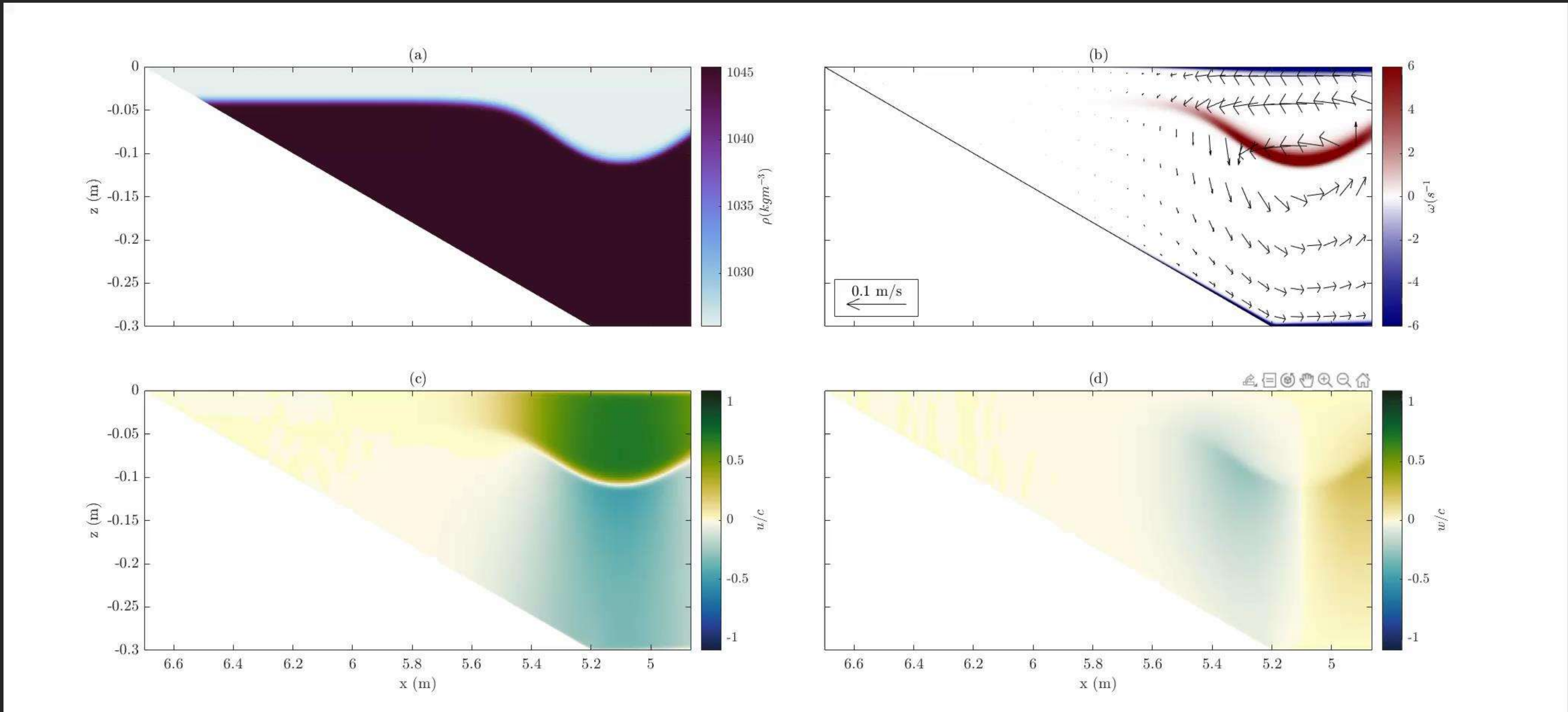
Shoaling Classification



(a) thin tanh profile, (b) surface stratification and (c) broad tanh profile

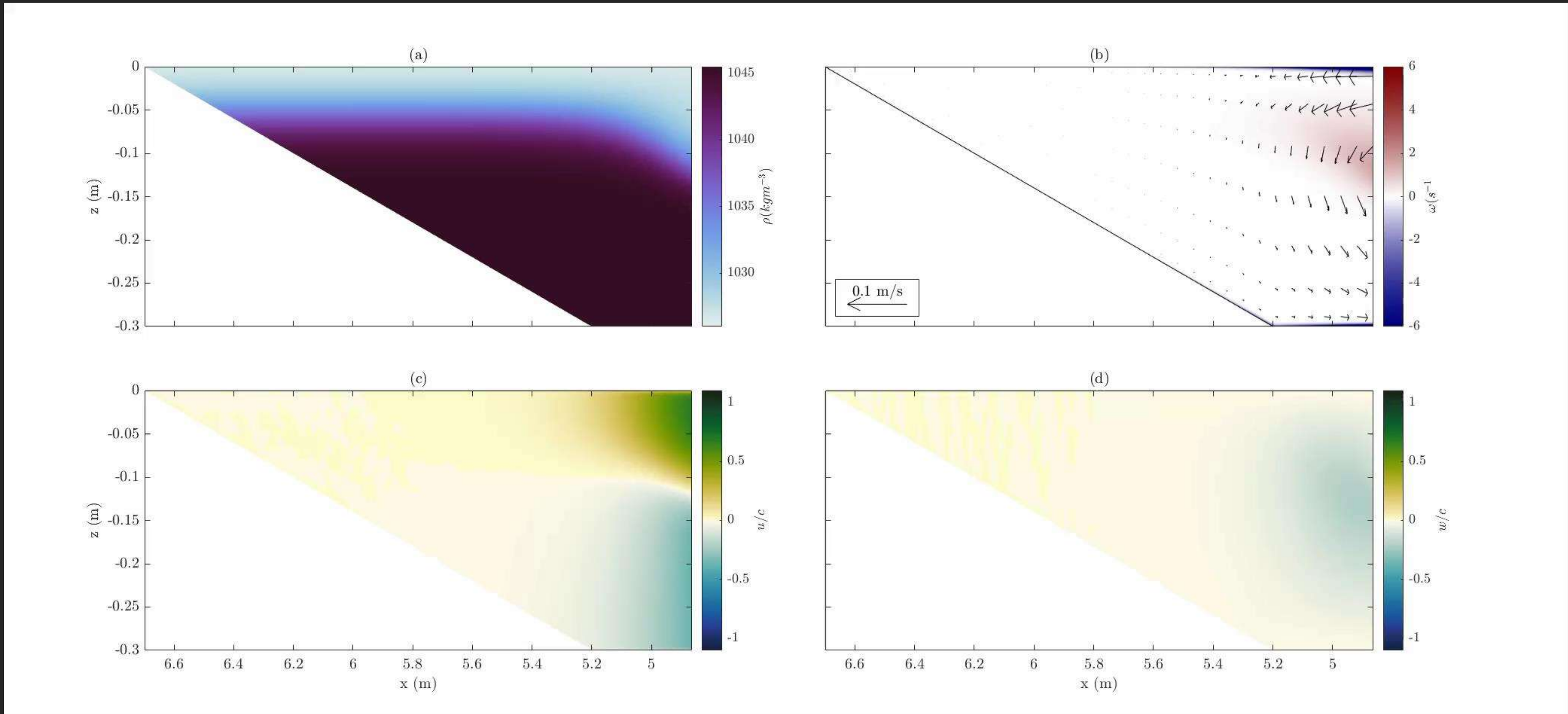
(+) Aghsaee et al., 2010; *J. Fluid Mech.*
(x) Nakayama et al., 2019; *Phys. Rev. Fluids.*
Filled circle - our numerical, Filled squares - our laboratory

Plunging example (thin tanh profile - 8L)



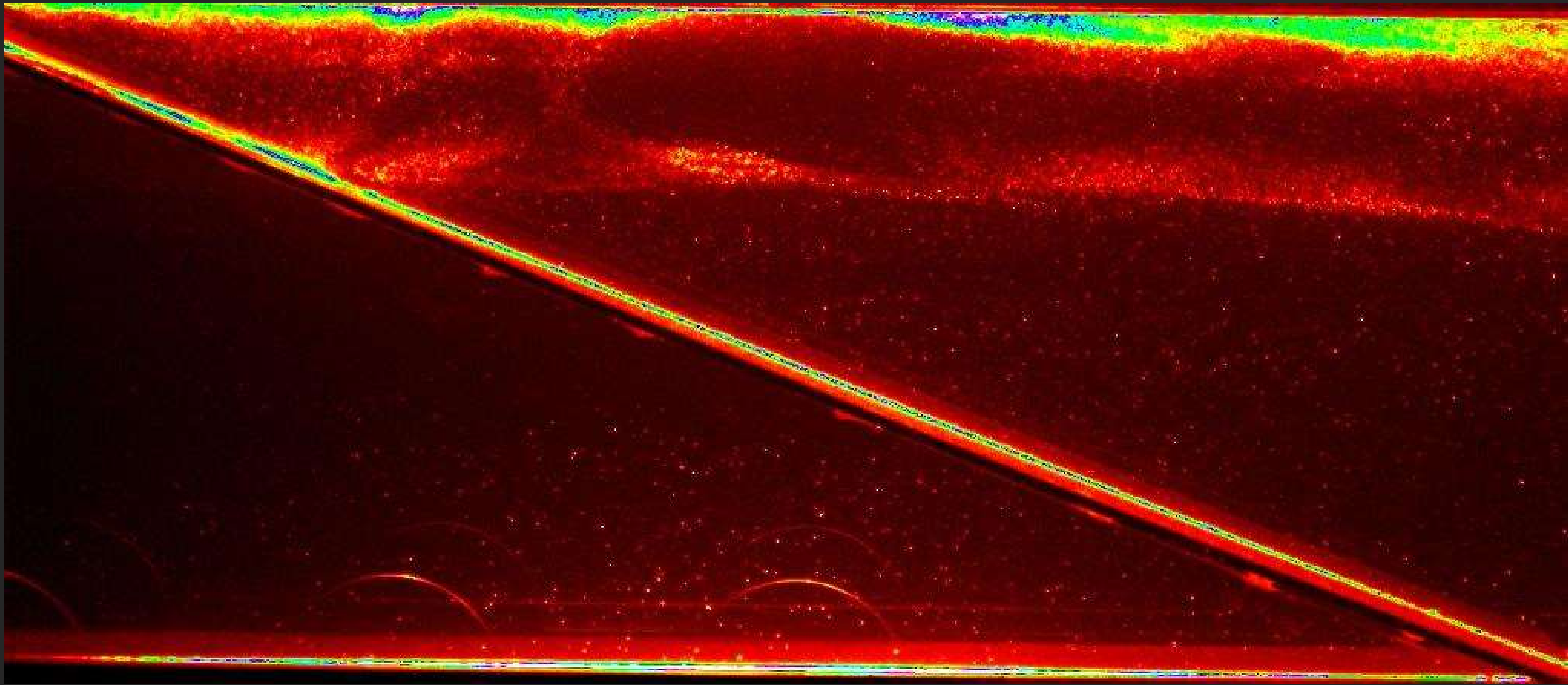
- Overturning leeward face becomes gravitationally unstable

Collapsing example (surface stratification - 7L)



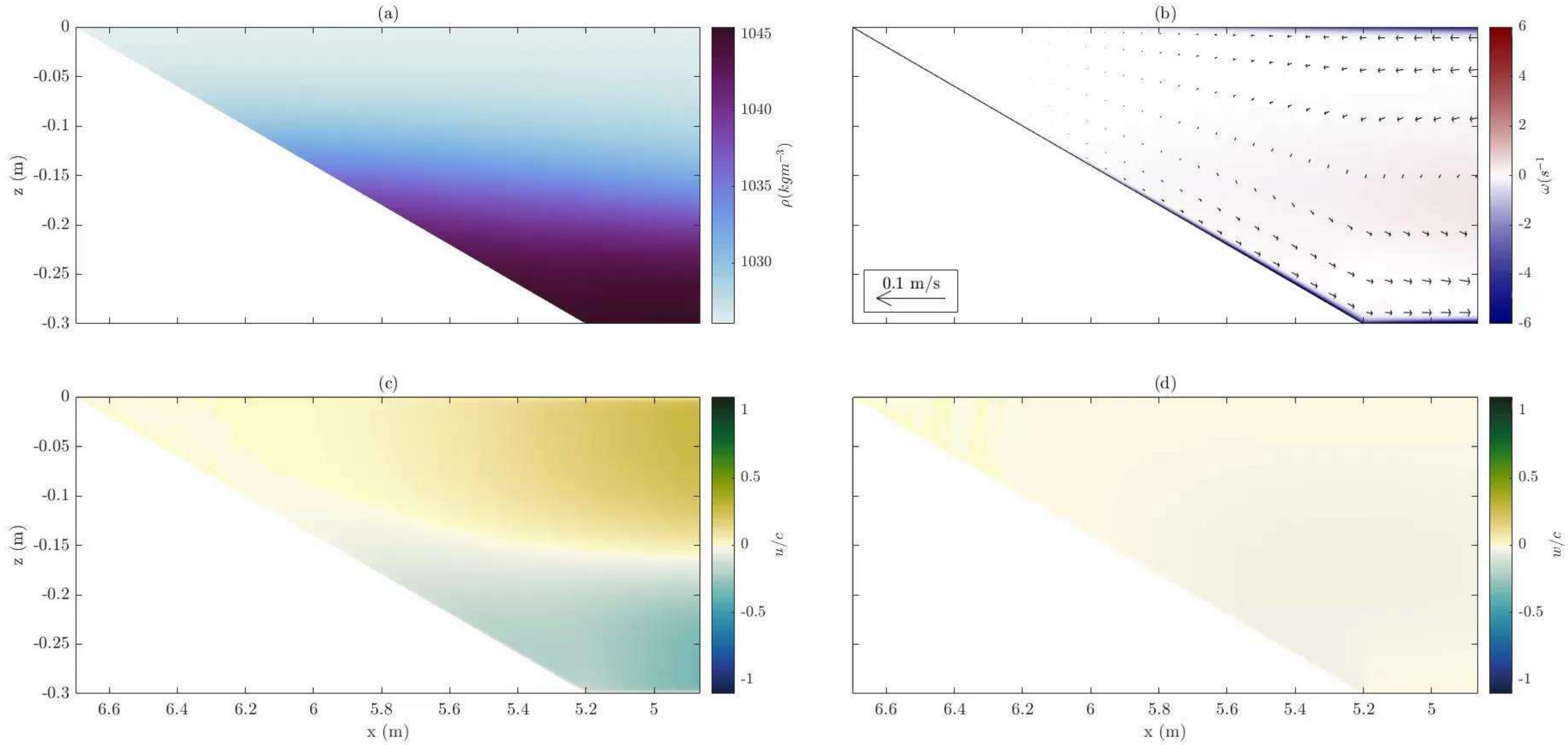
- Separated flow pushes the leeward face backward upon itself
- Density gradient in upper layer suppresses plunging

Collapsing example - laboratory



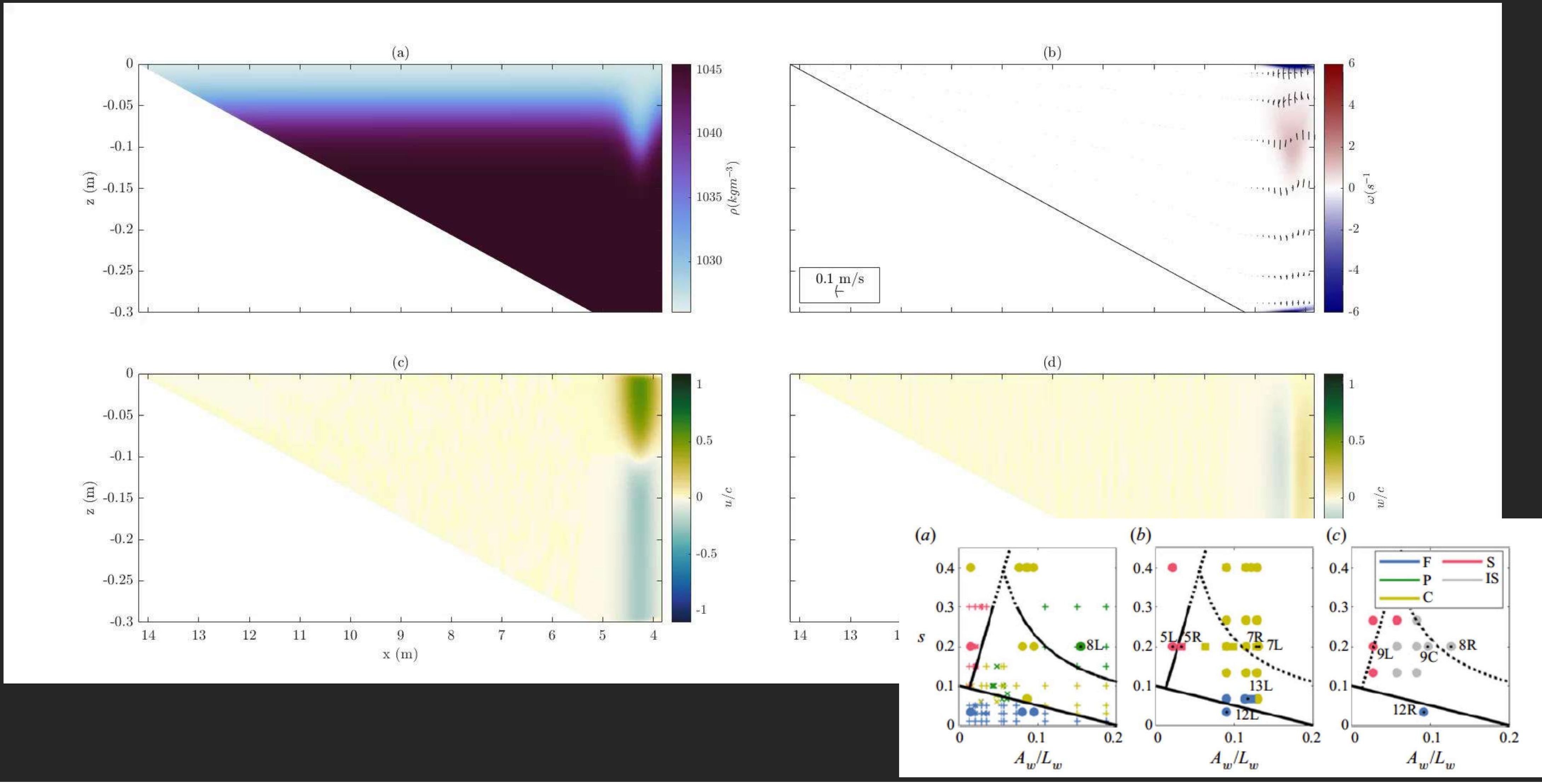
Slope height = 0.6m, Generating Volume = 30l

Surging example from broad tanh profile - 8R



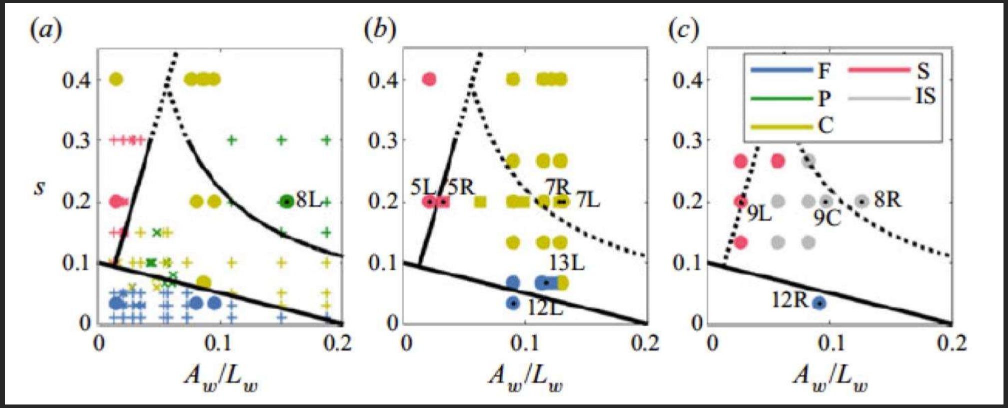
- Density gradient in lower layer suppresses BL vortex formation

Fission example (Surface stratification - 12L)



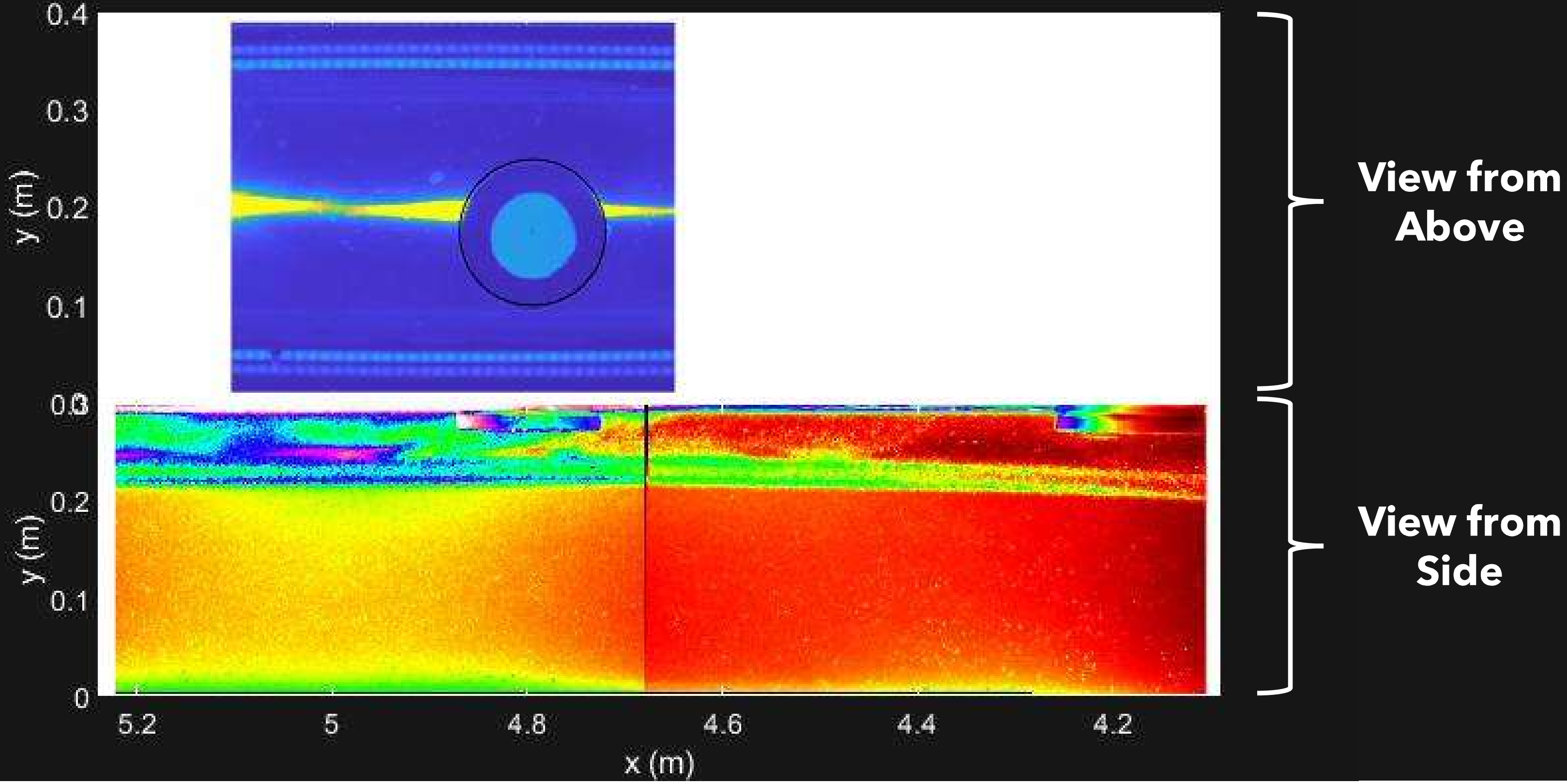
Main Findings

- Stratification form affects shoaling classification
- Density gradient in the upper layer suppresses plunging dynamics
- Density gradient in the lower layer suppresses collapsing dynamics
- Surging boluses can support K-H instabilities
- On slope steepness representative of continental slopes (s=0.03-0.07) we expect fission regardless of the stratification form.



Hartharn-Evans, Carr, Stastna & Davies. JFM 2022.

Ice Motion



0.4
0.3
0.2
0.1
y (m)
0.3
0.2
0.1
0
y (m)
5.2
5
4.8
4.6
4.4
4.2
x (m)
View from Above
View from Side

Equation for Ice Motion

$$F_D = \frac{1}{2}\rho_1 v^2 C_d A$$

$$v = u_x^t - U_{ice}^t$$

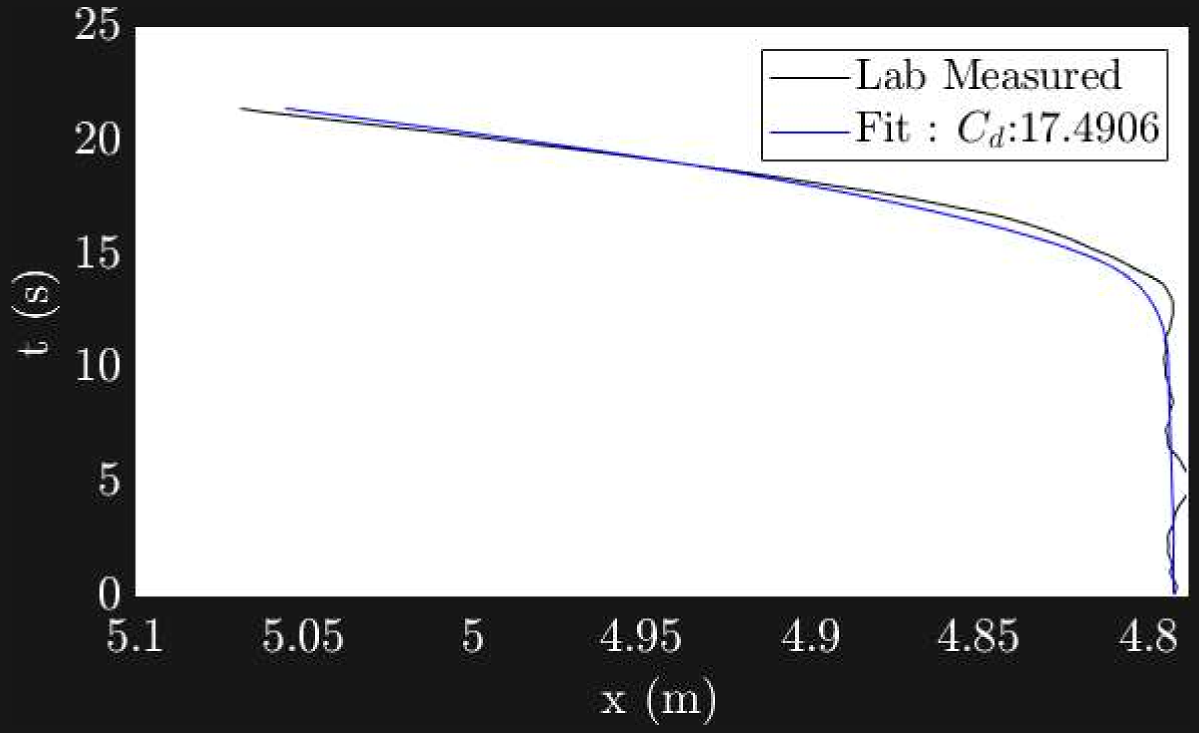
$$a = \frac{F_D}{m}$$

$$a_{ice}^t = \frac{1}{2} C_d \frac{\rho_1}{\rho_f} \left(u_x^t - U_{ice}^t\right)\left|\left(u_x^t - U_{ice}^t\right)\right|$$

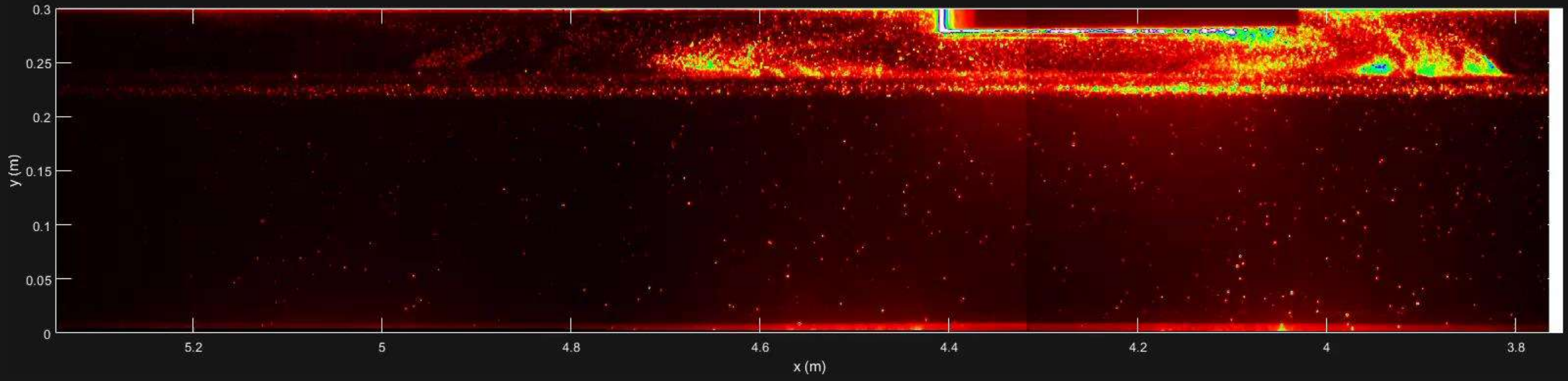
$$U_{ice}^{t+1} = U_{ice}^t + \Delta t \, a_{ice}^t$$

$$X_{ice}^{t+1} = X_{ice}^t + \Delta t \, U_{ice}^t$$

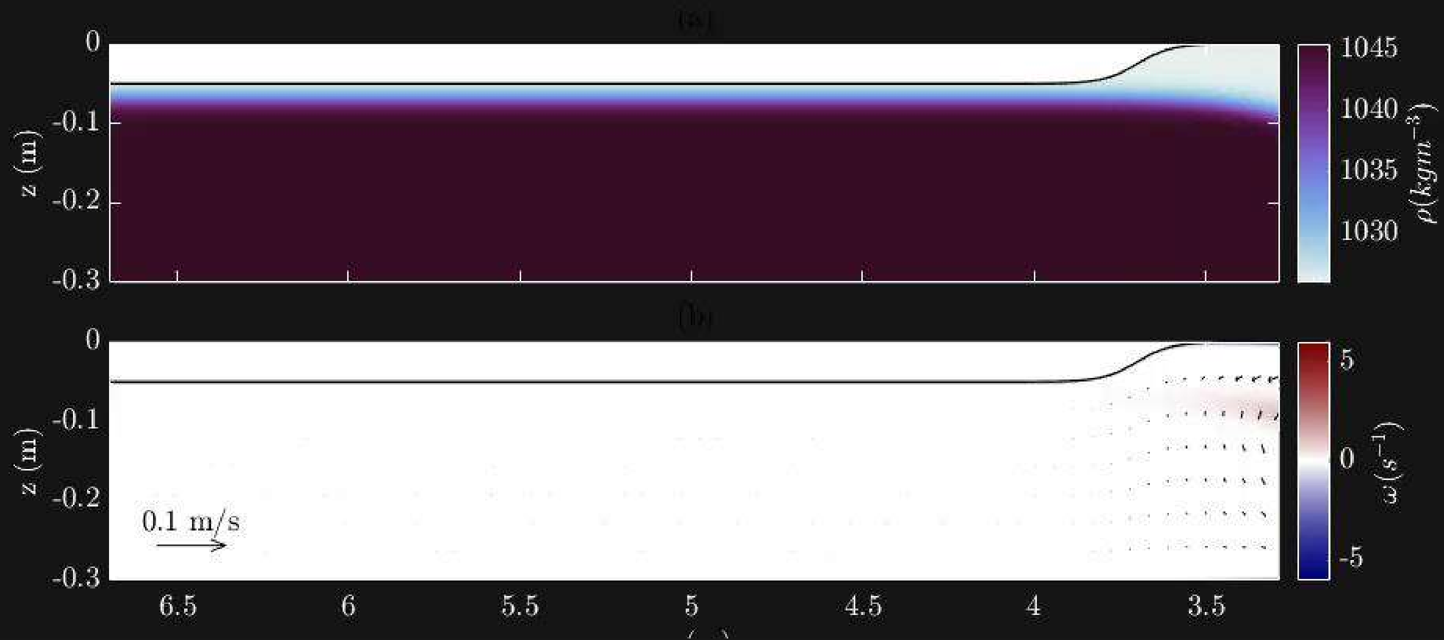
Calculated Drag Coefficient



Large "Ice" Motion

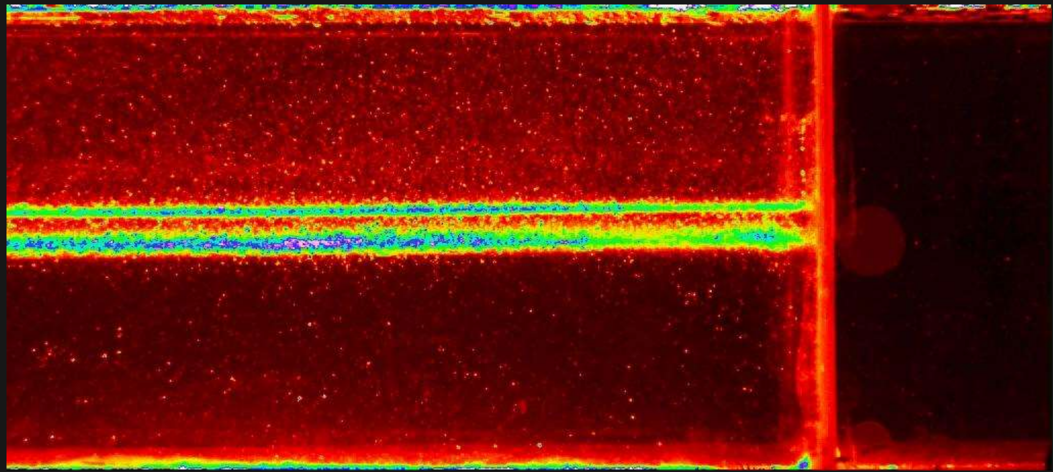


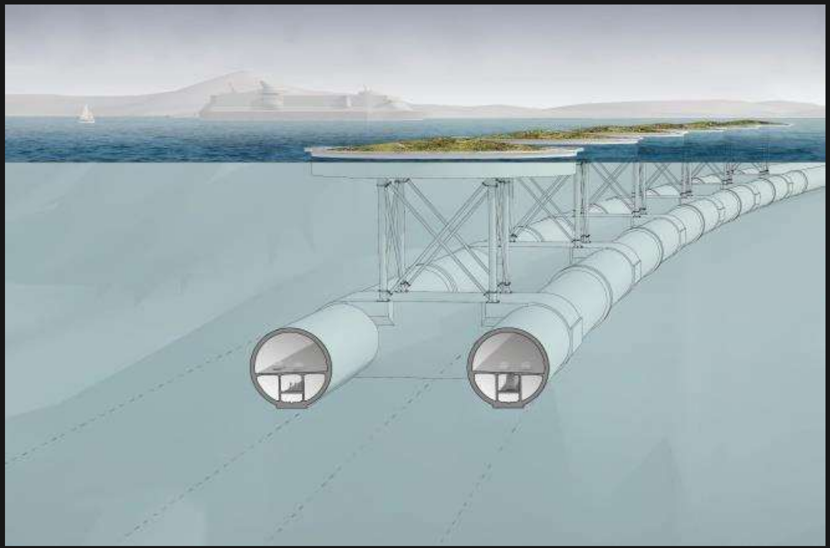
Numerical Wave/Ice Collisions



Future interests

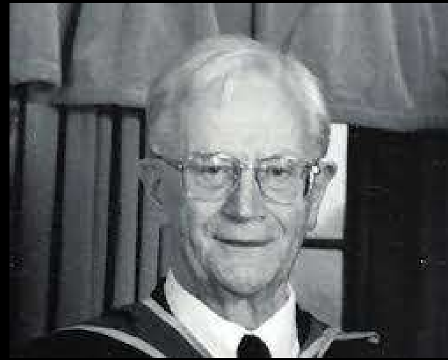
- Mode 2 ISWs
- Offshore wind
- Floating/submerged bridges
- Under water communications





A bit about me

- BSc in Mathematics with European Studies: Durham University, 1995-1999
- Industrial Engineer: Viasystems Ltd, South Shields. 1999-2000
- PhD in Applied Mathematics: Durham University, 2000-2003
- Postdoctoral Research Assistant: University of Dundee, 2003-2006
- Lecturer/Senior Lecturer: University of St Andrews, 2006-2018
- Senior Lecturer: Newcastle University, 2018-present



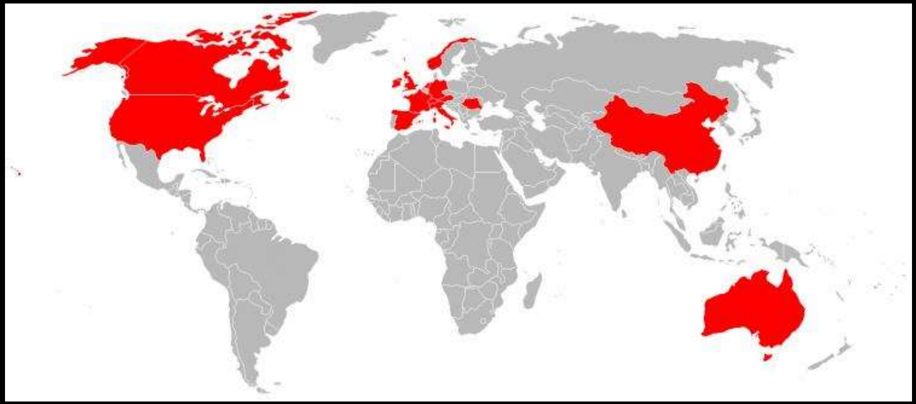
Dr Vernon Armitage

Prof Brian Straughan

Prof Peter. A. Davies

Why have I stayed in academia ?

- World Expert in the field - highly specialised & original work.
- Opportunity to travel the world - meet people, discuss & present work, share your knowledge.



- Variety - Research, Teaching, Administration.
- Flexibility - Own boss. Hybrid working.
- Reflections – The times they are a changing, thanks to Piscopia and other initiatives.