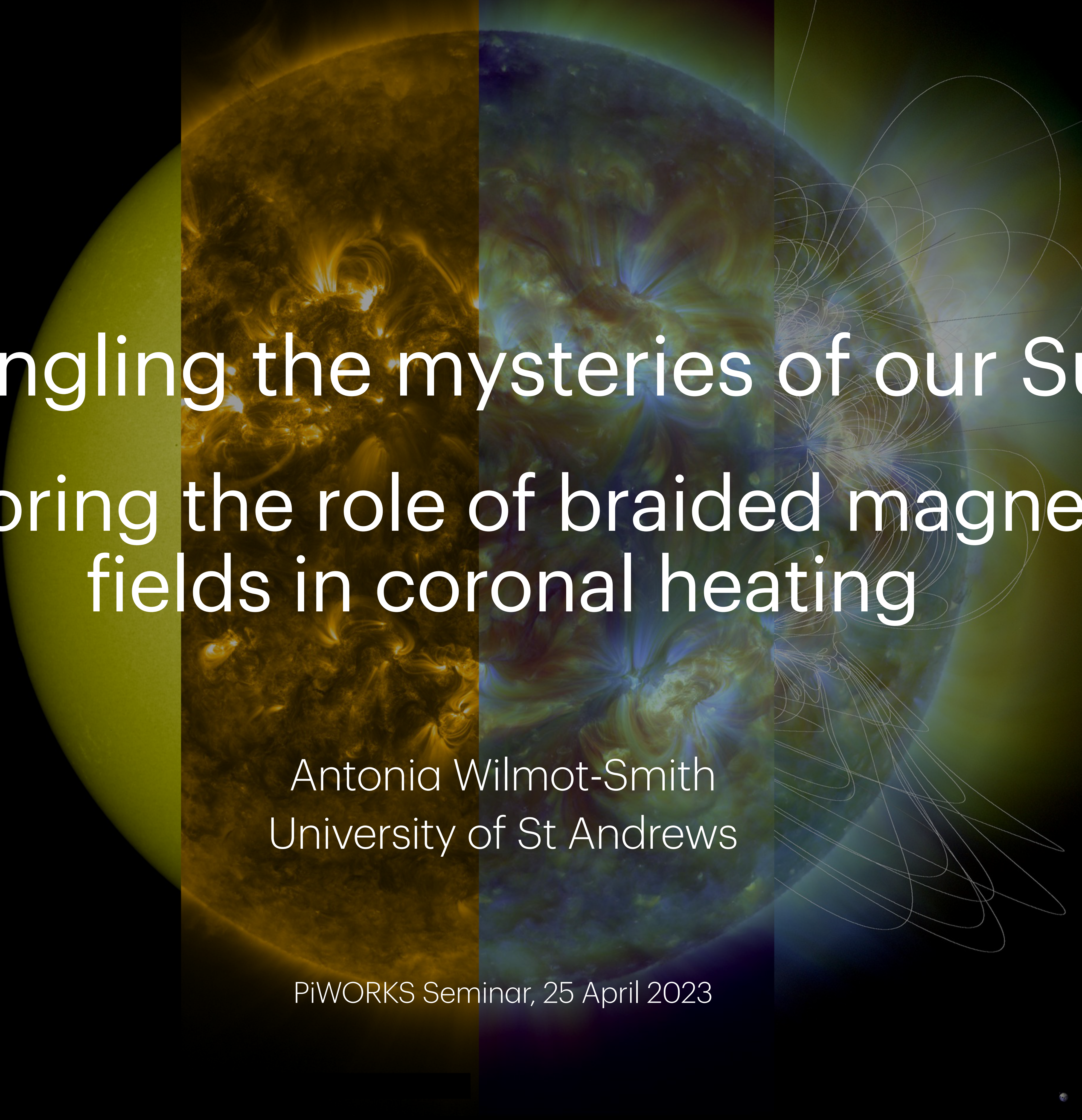
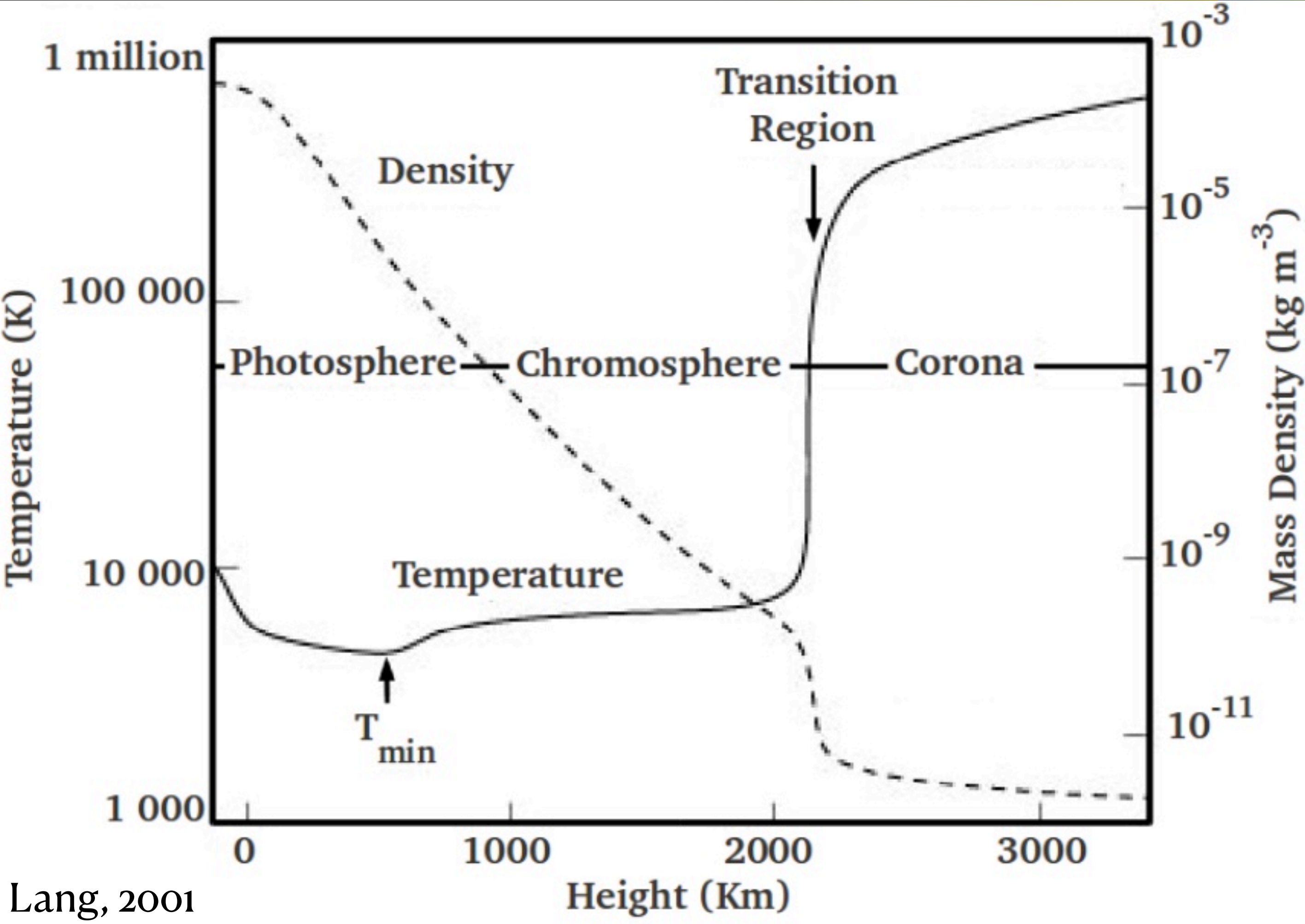


# Untangling the mysteries of our Sun:

# Exploring the role of braided magnetic fields in coronal heating

Antonia Wilmot-Smith
University of St Andrews

PiWORKS Seminar, 25 April 2023




Lang, 2001

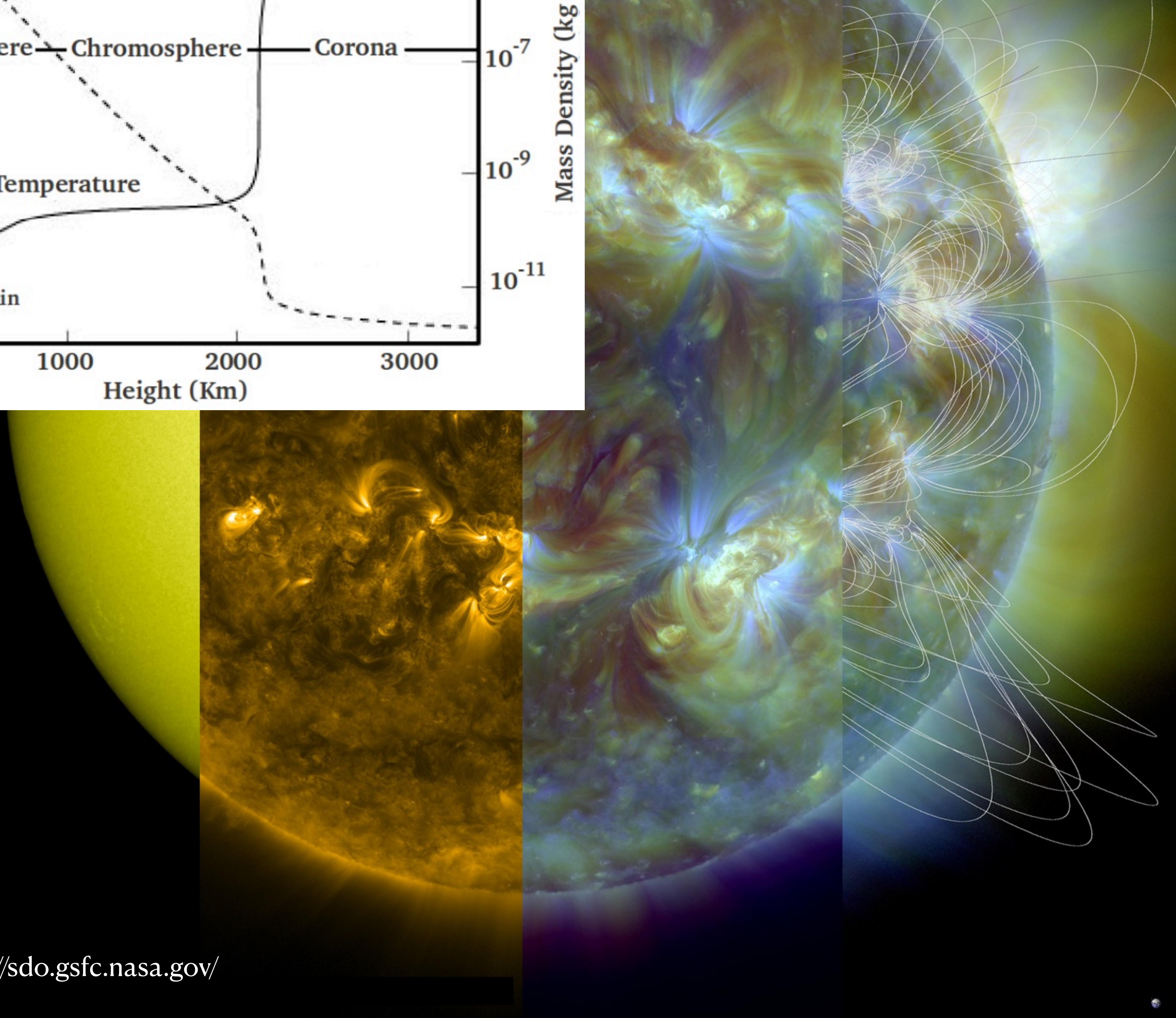
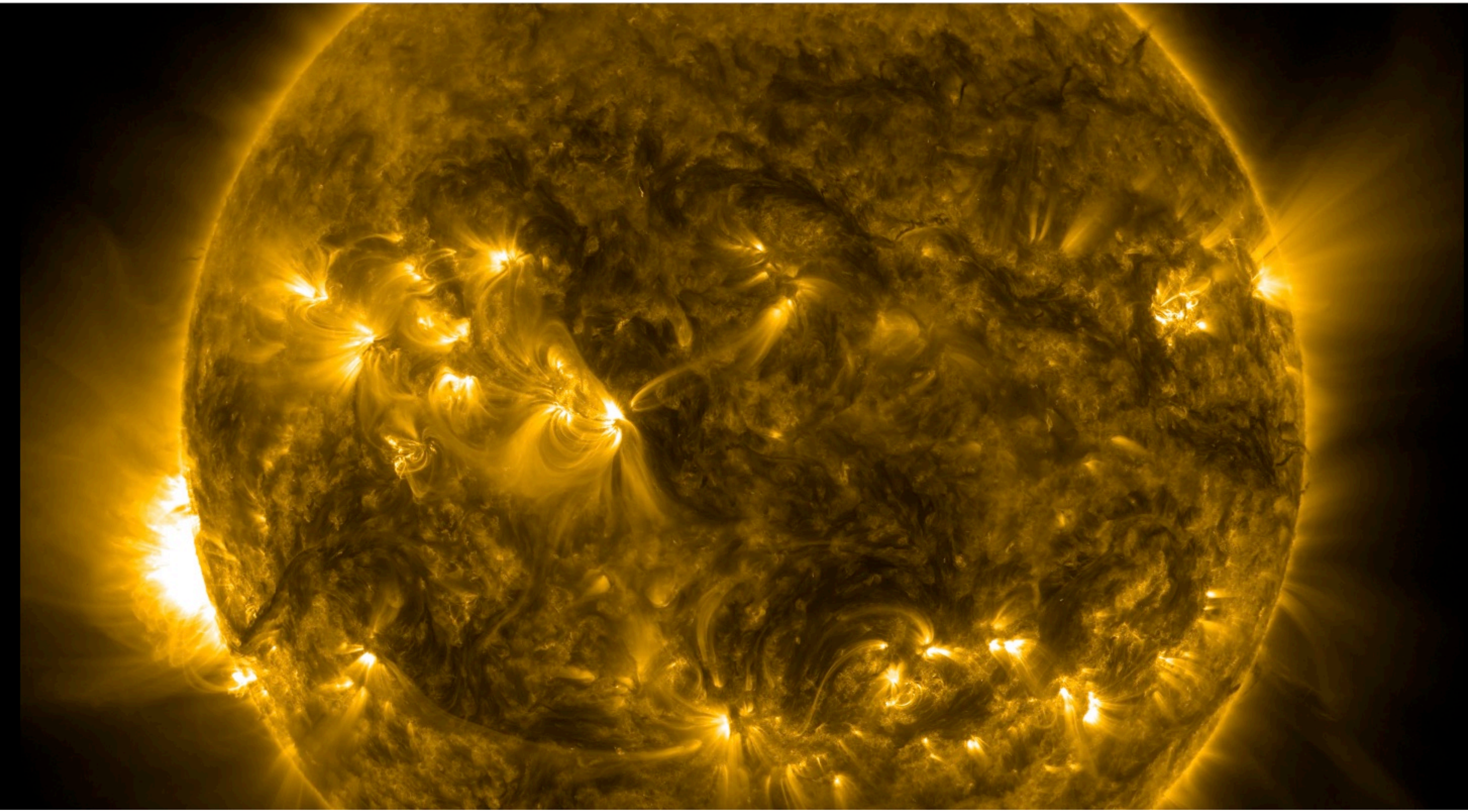


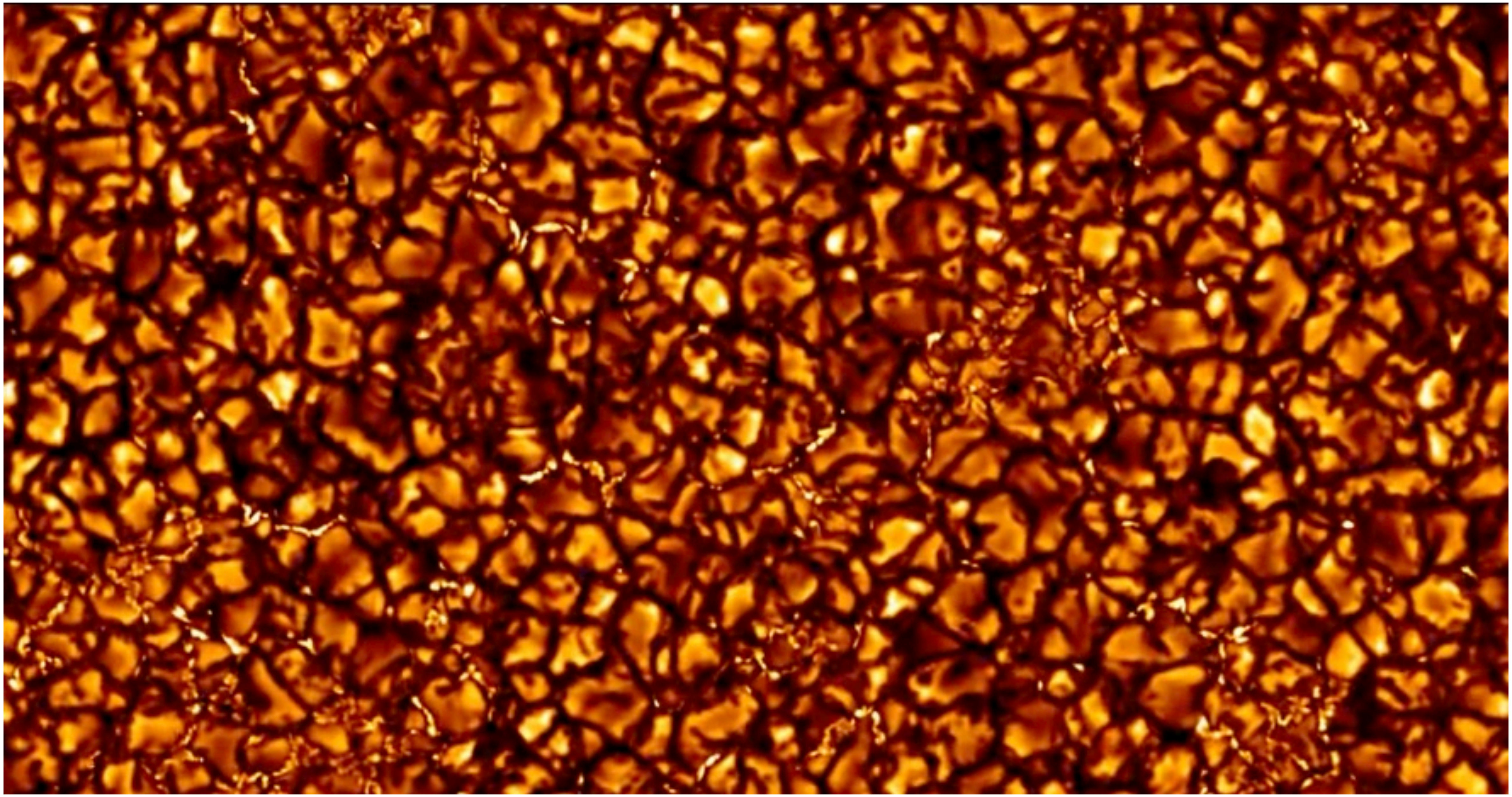
image credit: https://sdo.gsfc.nasa.gov/

# Dynamic Corona



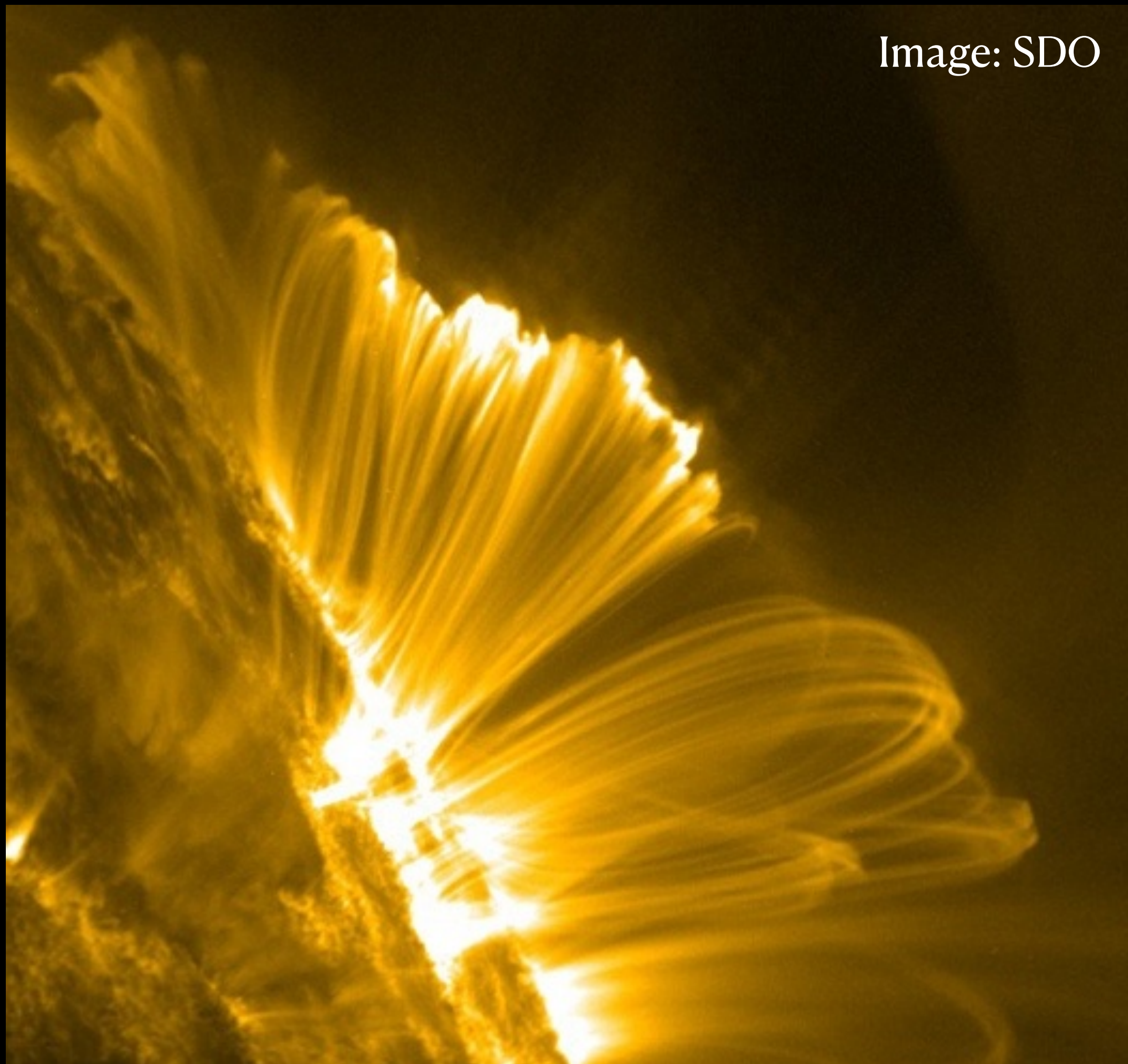
Video: https://sdo.gsfc.nasa.gov/assets/img/ultra_hd/0171304Whip_best.mp4

# Photospheric Motions



Movie: https://commons.wikimedia.org/wiki/File:Granulation_Quiet_Sun_SST_25May2017.webm

Credit: Swedish Solar Telescope



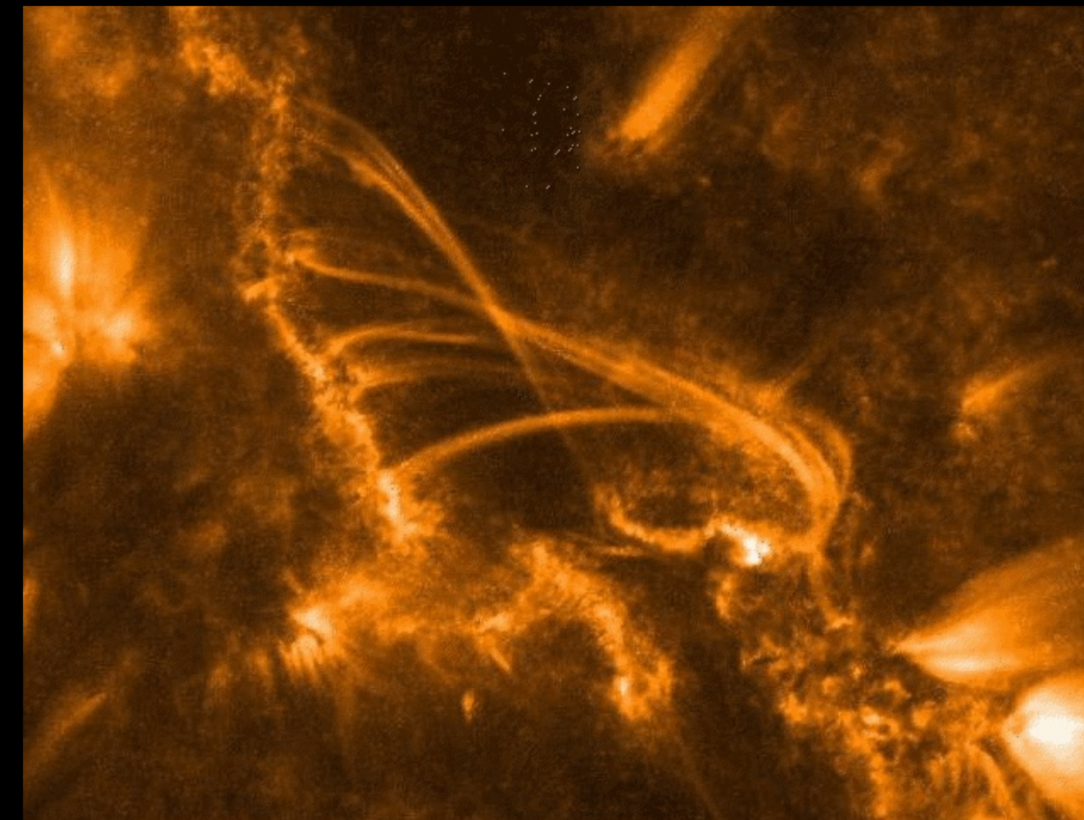



Image: TRACE satellite

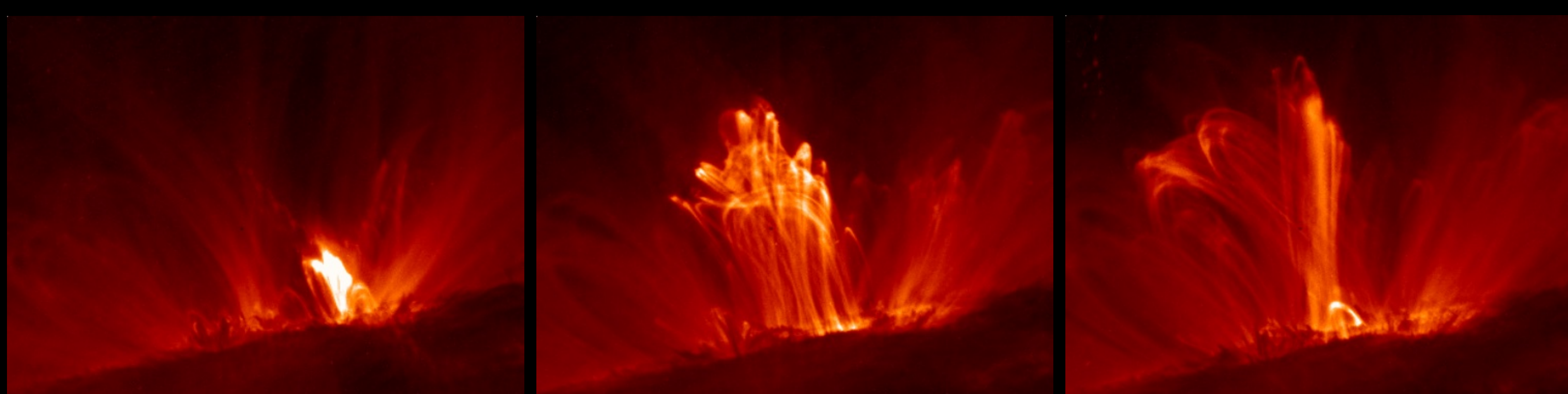


Image: TRACE satellite

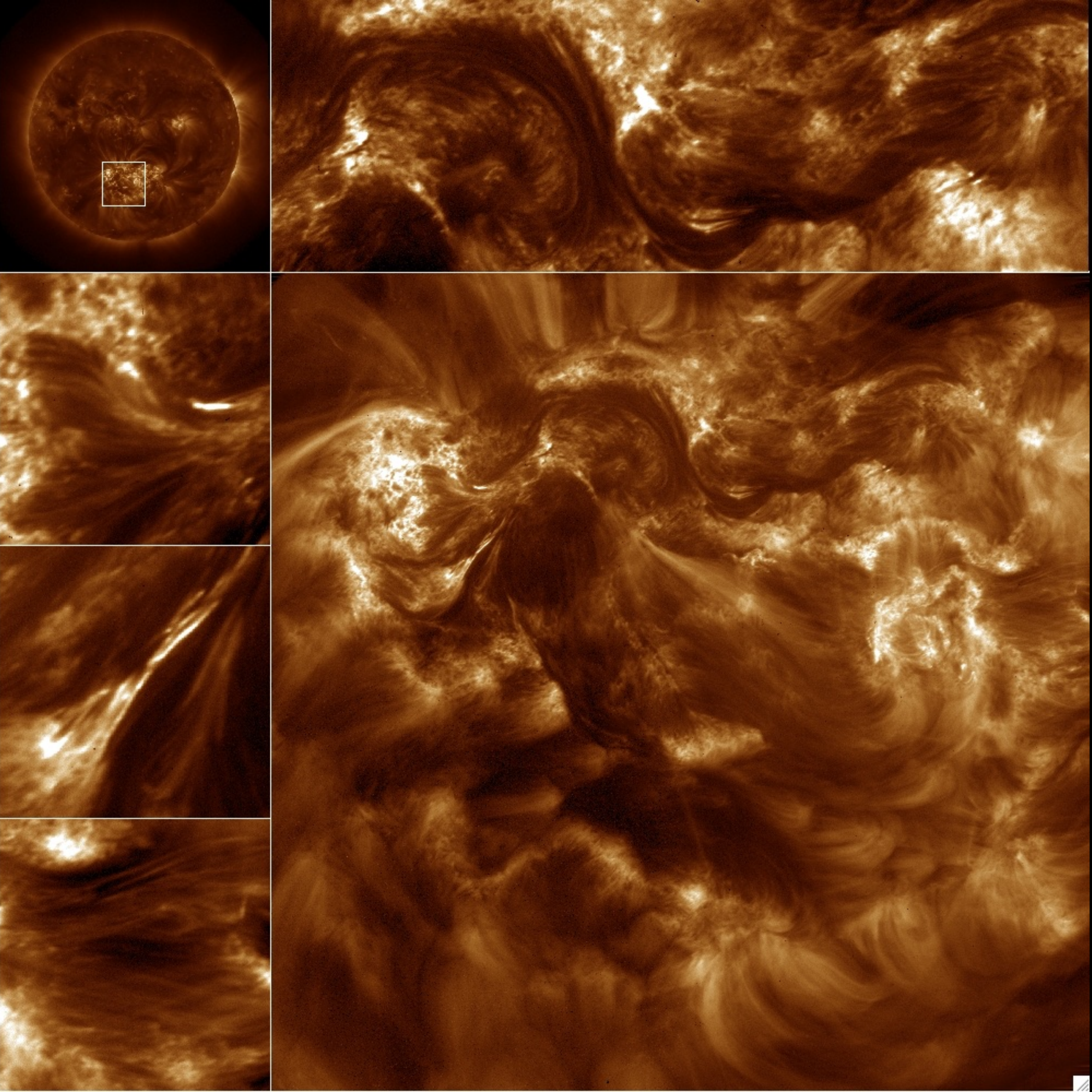


Image: NASA, Hi-C

# MHD modelling

Continuity equation: $$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \mathbf{v}\right) = 0$$

density; plasma velocity

Equation of motion: $$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \mathbf{j} \times \mathbf{B} + \rho\, \mathbf{g}$$

pressure; magnetic field; current density; gravitational force

Ampère's law: $\nabla \times \mathbf{B} = \mu \mathbf{j}$

Faraday's law: $$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

electric field

Solenoidal constraint: $\nabla \cdot \mathbf{B} = 0$

Ohm's law: $\mathbf{j} = \sigma\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$

+ Energy equation + Ideal gas law

Suggested resource: An introduction to Magnetohydrodynamics, P.A. Davidson, CUP 2001.

# Induction Equation

- Use Ohm's law, Ampère's law and Faraday's law to eliminate **E** (the electric field), take $\eta = 1/(\mu\sigma)$ as a constant, use vector identities and the solenoidal constraint to get

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

- In MHD **v** (the velocity) and **B** (the magnetic field) are the primary variables
- The induction equation is an advection-diffusion equation for **B** (the magnetic field).

Suggested resource: Magnetohydrodynamics of the Sun, E. R. Priest, CUP (2014)

# Ideal evolution & Frozen-In Flux

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

Alfvén's theorem (Hannes Alfvén, 1943):

In a perfectly conducting fluid, magnetic field lines move with the fluid: the field lines are 'frozen' into the plasma.

- Motion along magnetic field lines doesn't change their topology
- Motions transverse to the magnetic field lines carry the magnetic field lines with them

Suggested resource: Magnetohydrodynamics of the Sun, E. R. Priest, CUP (2014)

# Force-Free Fields

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \mathbf{j} \times \mathbf{B} + \rho \mathbf{g}$$

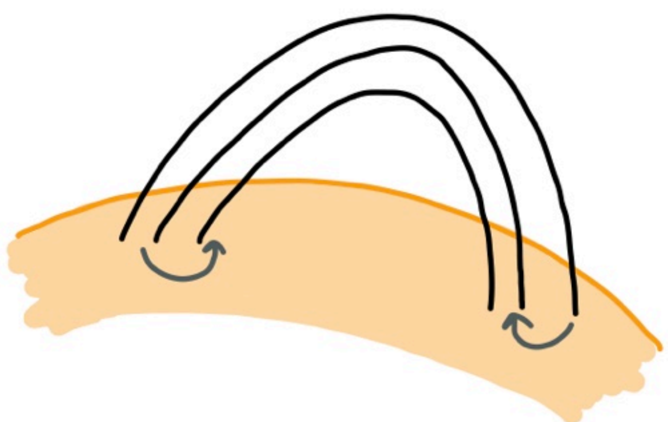
# Force-Free Fields

$$(\nabla \times \mathbf{B}) \times \mathbf{B} = \mathbf{0}$$

- In cases where $\mathbf{B} \neq \mathbf{0}$ in the domain this is equivalent to $\nabla \times \mathbf{B}(x) = \alpha(x)\mathbf{B}(x)$ (*Beltrami fields* of fluid dynamics)
- Since $\nabla \cdot \mathbf{B} = 0$ we also have $(\mathbf{B} \cdot \nabla)\,\alpha = 0$ i.e. $\alpha$ is constant along magnetic field lines
- Case of $\alpha =$ constant in the domain is known as a *linear force-free field*

Suggested resource: *Solar force-free magnetic fields*, Wiegelmann & Samurai, LRSP **18** 1 (2021)

# Physical scenario



Photospheric motions act on footprints of solar coronal loop

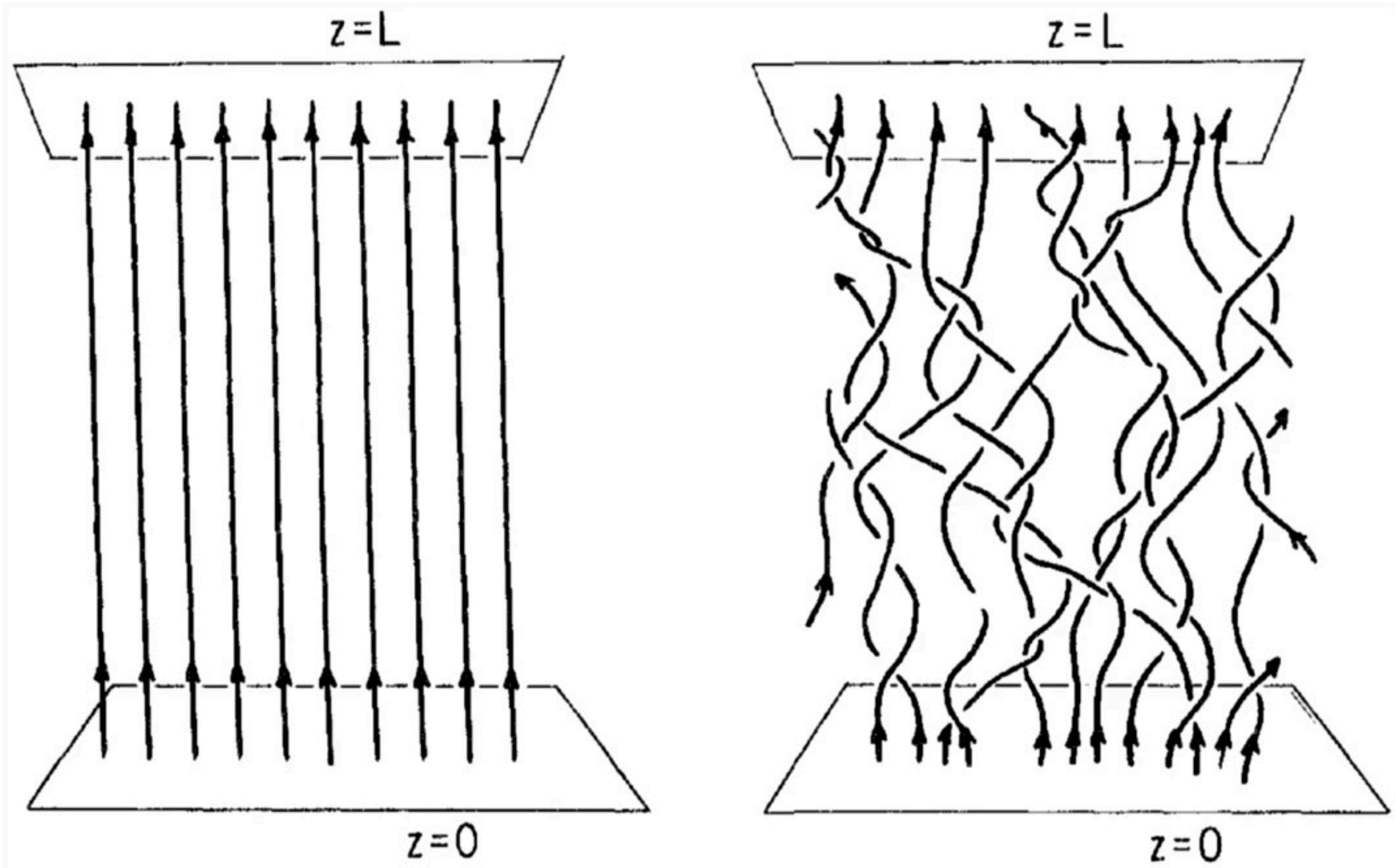
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

Magnetic field in the loop evolves ideally (frozen-in flux)

$$(\nabla \times \mathbf{B}) \times \mathbf{B} = \mathbf{0}$$

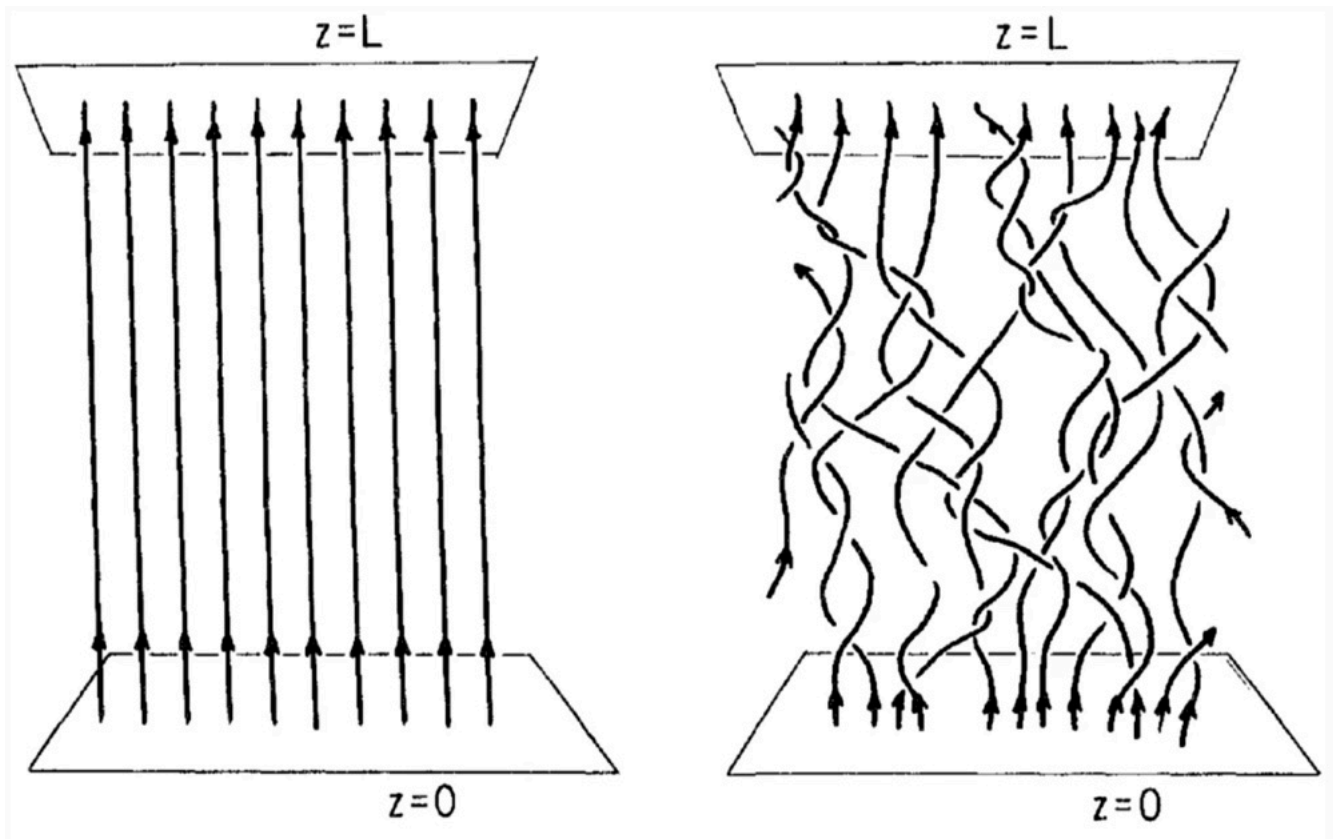
Magnetic field relaxes to a force-free equilibrium

# The Parker Problem




Parker, E.N. (1994) *Spontaneous current sheets in magnetic fields: with applications to stellar X-rays*
International Series in Astronomy and Astrophysics, vol 2. Oxford University Press, Oxford

# The Parker Problem

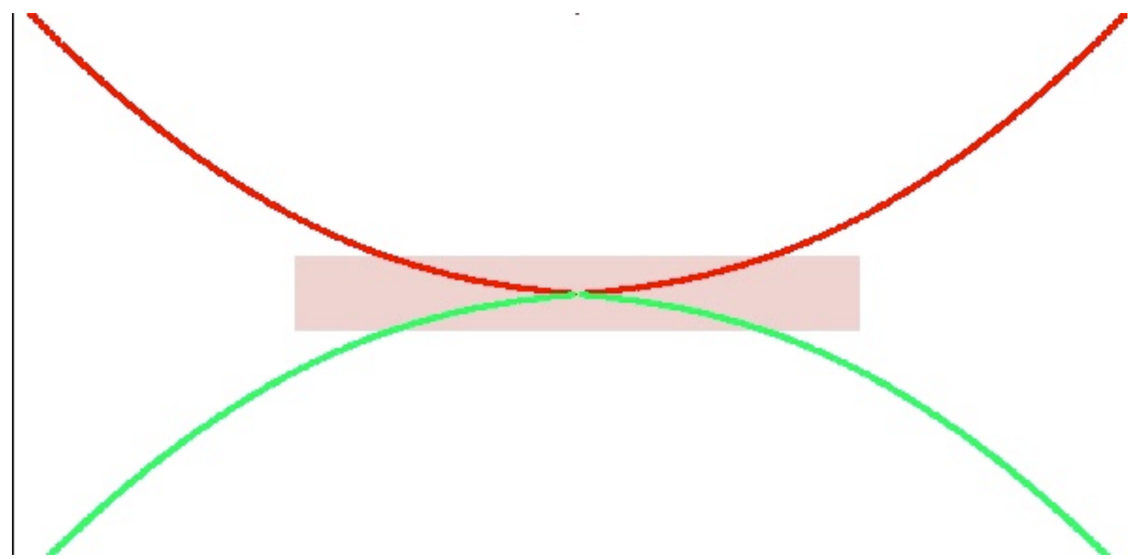



Given an arbitrary flow on the boundary of the domain, can the magnetic field relax to a smooth force-free equilibrium or do tangential discontinuities develop in the magnetic field?

Suggested resource: *The Parker problem: existence of smooth force-free fields and coronal heating*, Pontin & Hornig, Living Reviews in Solar Physics **17** 5 (2020)

# Current sheets and magnetic reconnection

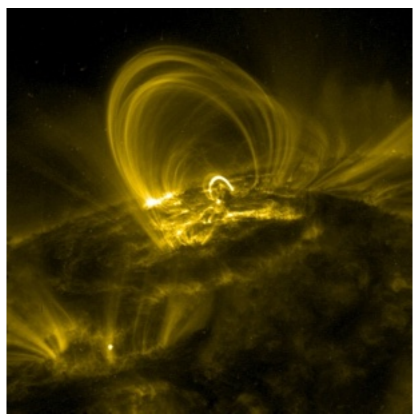
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

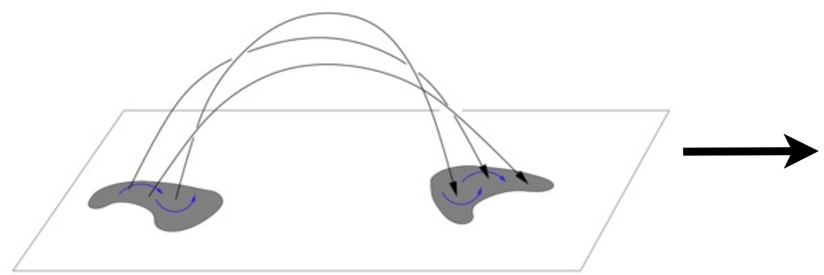


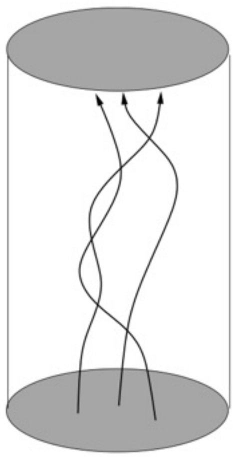
- Diffusion important in current sheets & leads to changes in connectivity of the magnetic field, releasing magnetic energy

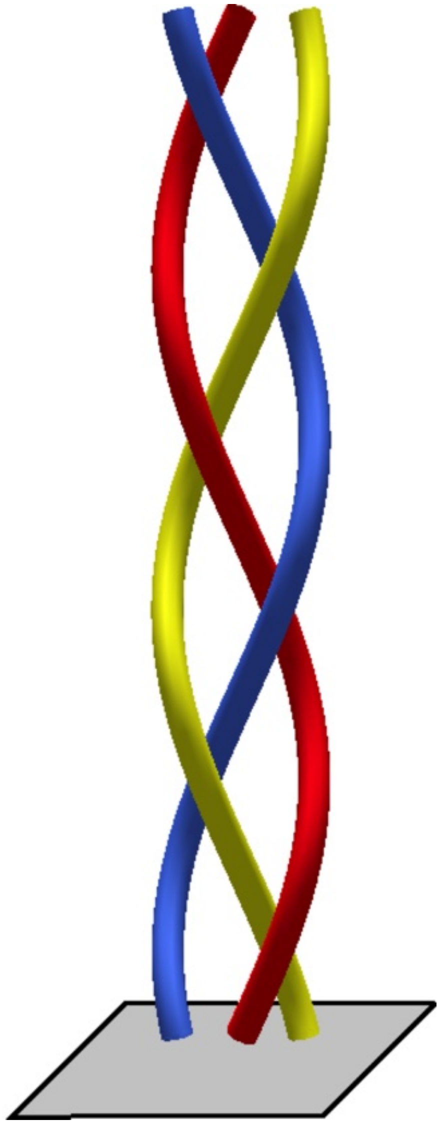
Suggested resource: *Magnetic Reconnection: MHD theory and modelling,* Pontin & Priest Living Reviews in Solar Physics **19** 1 (2022)

# Modelling Approach



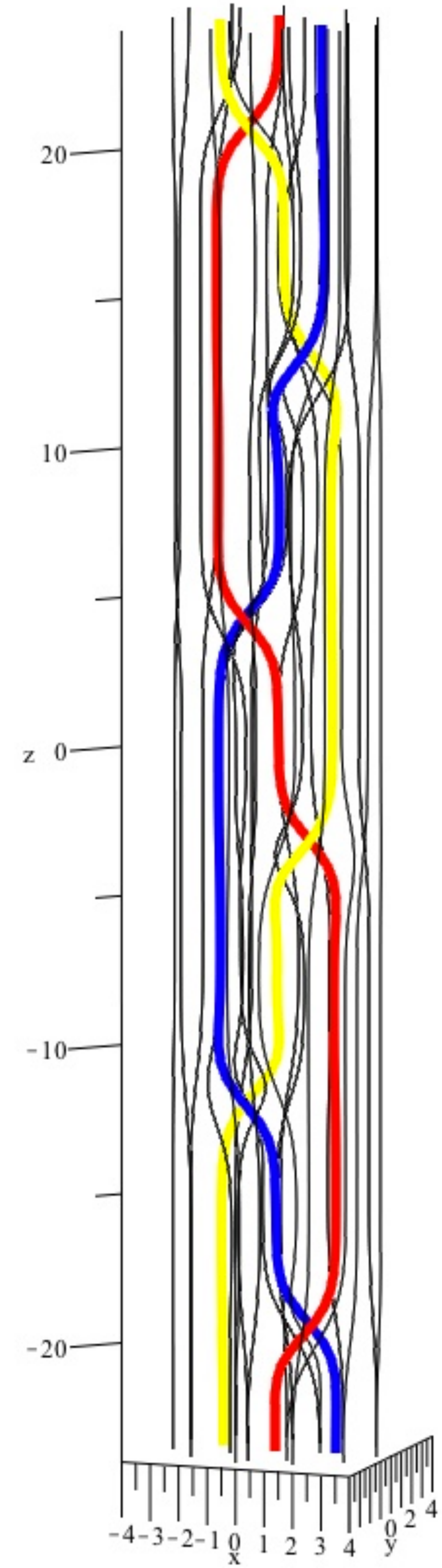






# Modelling Approach

- Start with an analytical magnetic field configuration based on pigtail braid
- Use a numerical code to relax the magnetic field towards a force-free state



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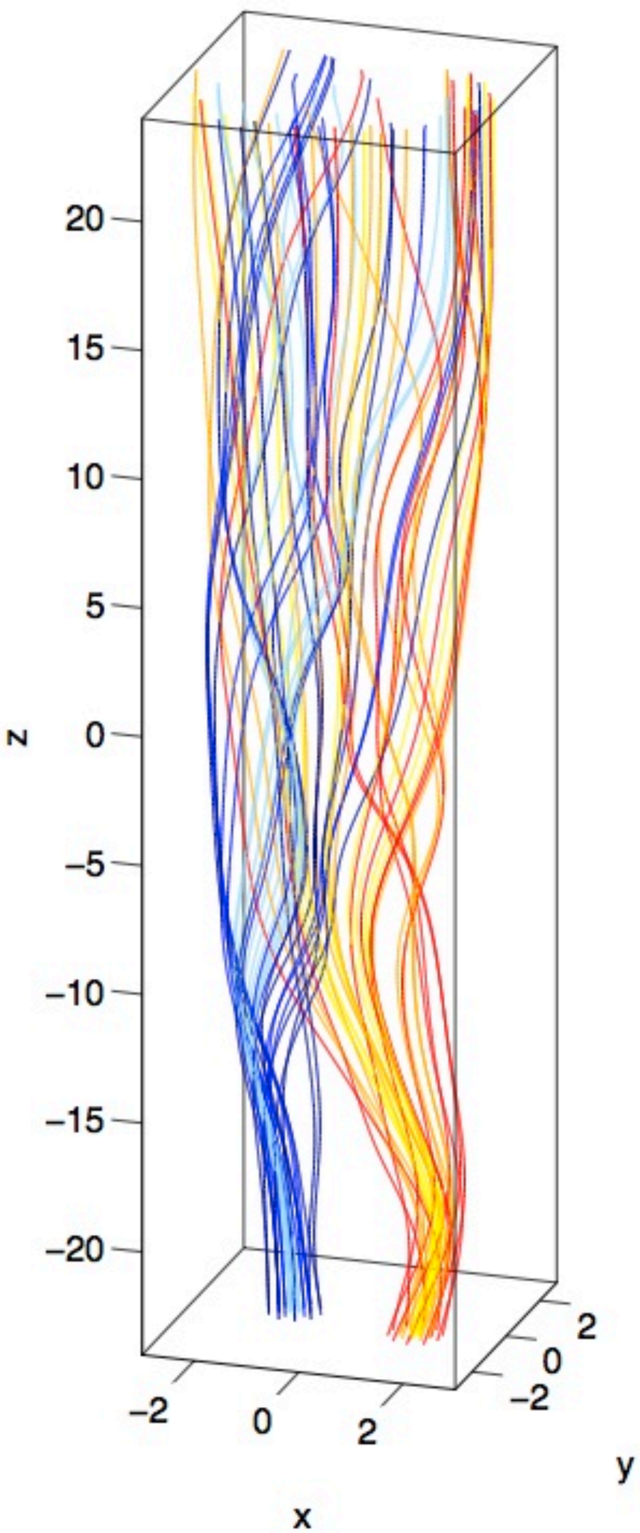
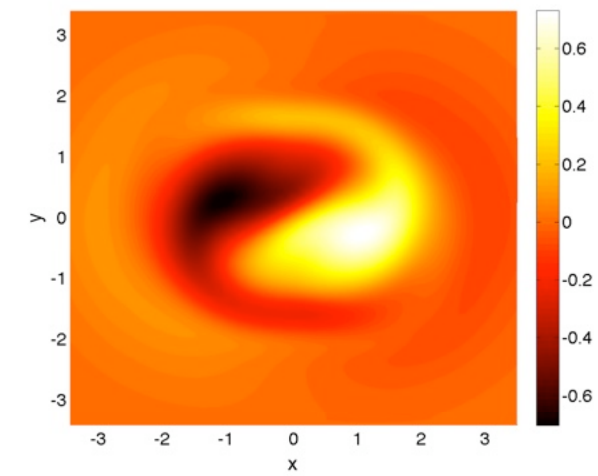
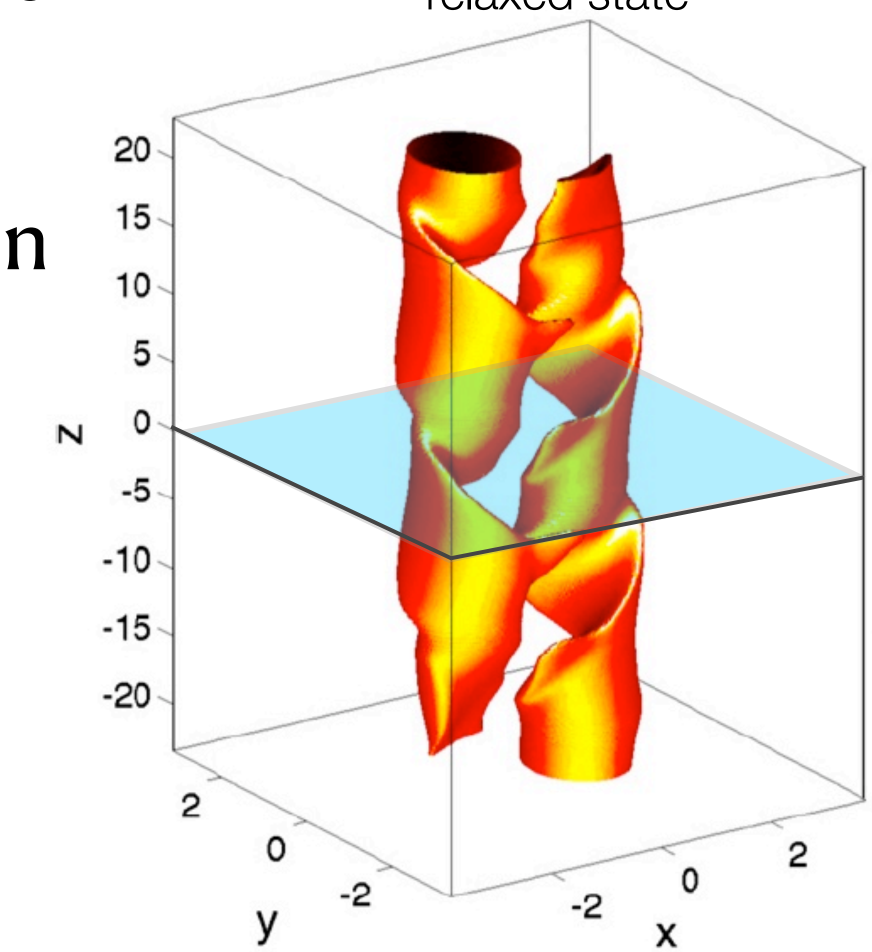



- Start with an analytical magnetic field configuration based on pigtail braid
- Use a numerical code to relax the magnetic field towards a force-free state
- Smooth variation of current in the ideal evolution
- BUT small scales develop in integrated quantities

$J_z$, z=0




Isosurface $|\mathbf{J}|_{max}/4$ in relaxed state



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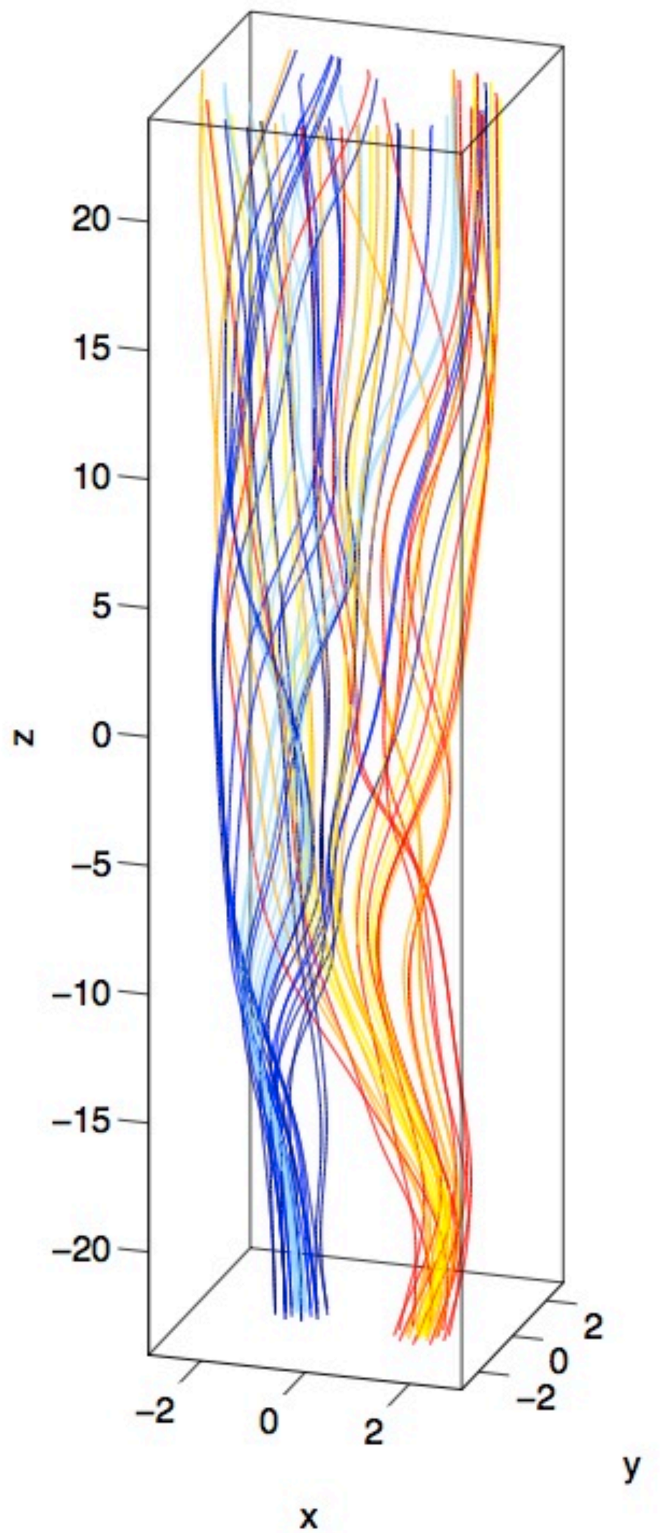


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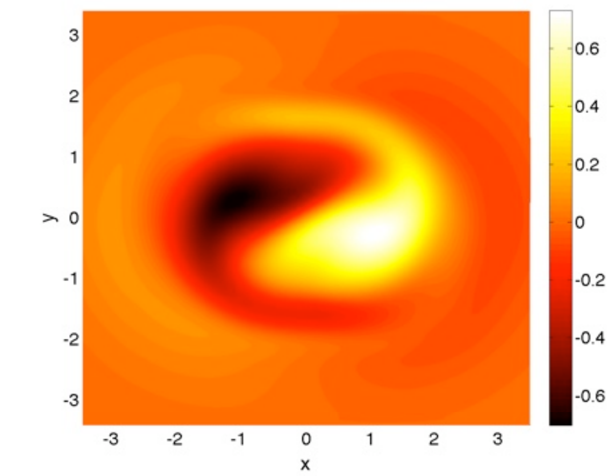


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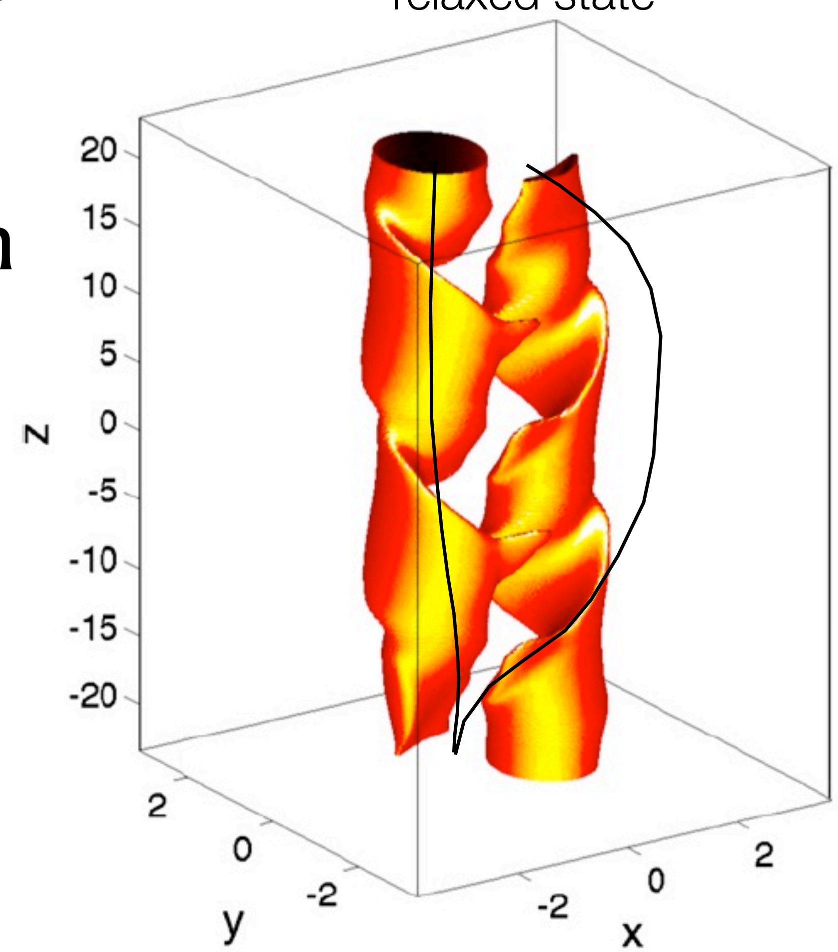

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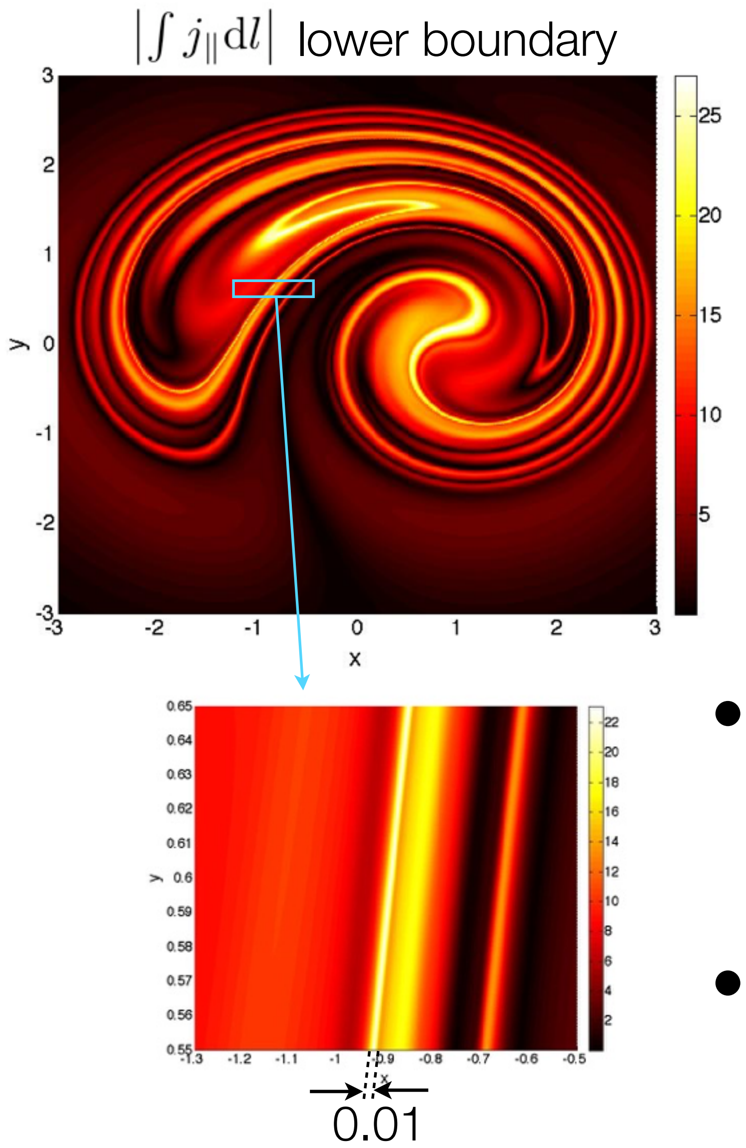


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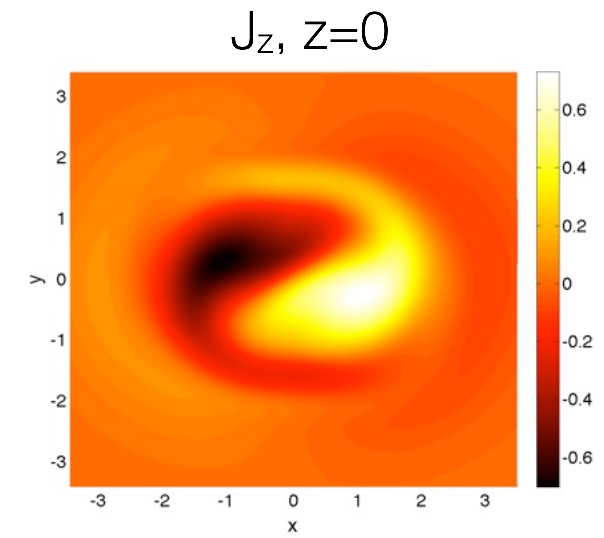
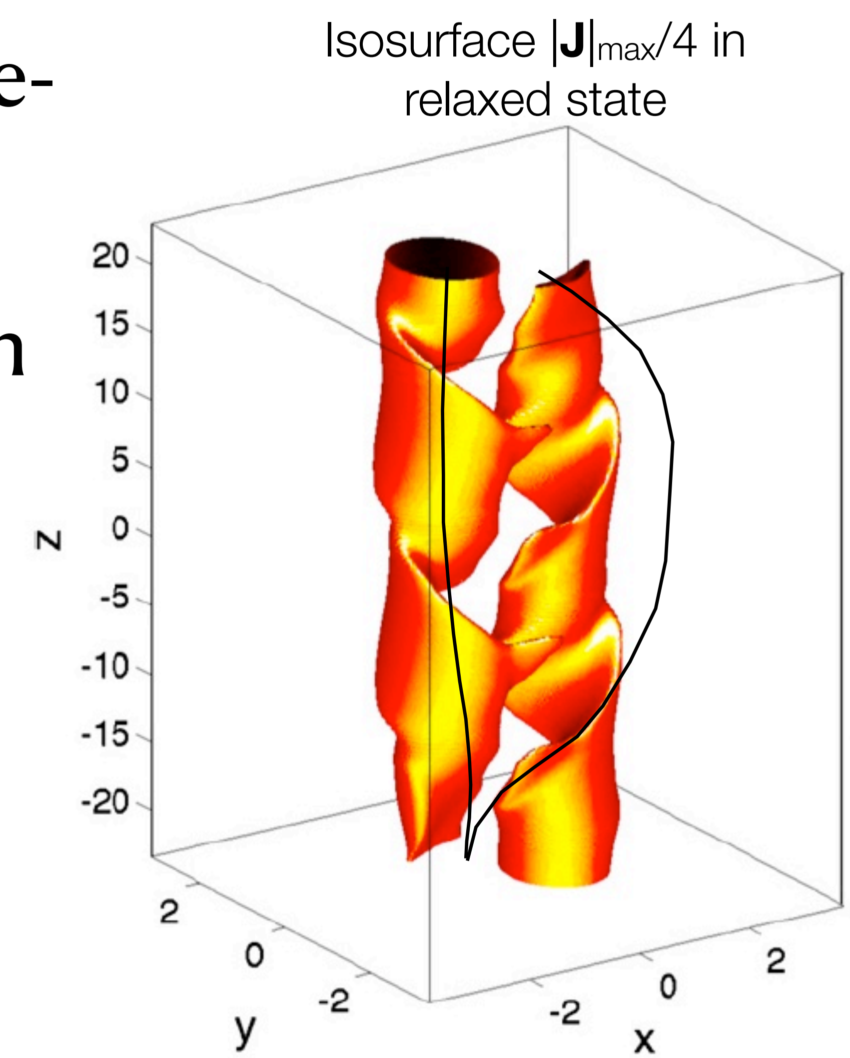
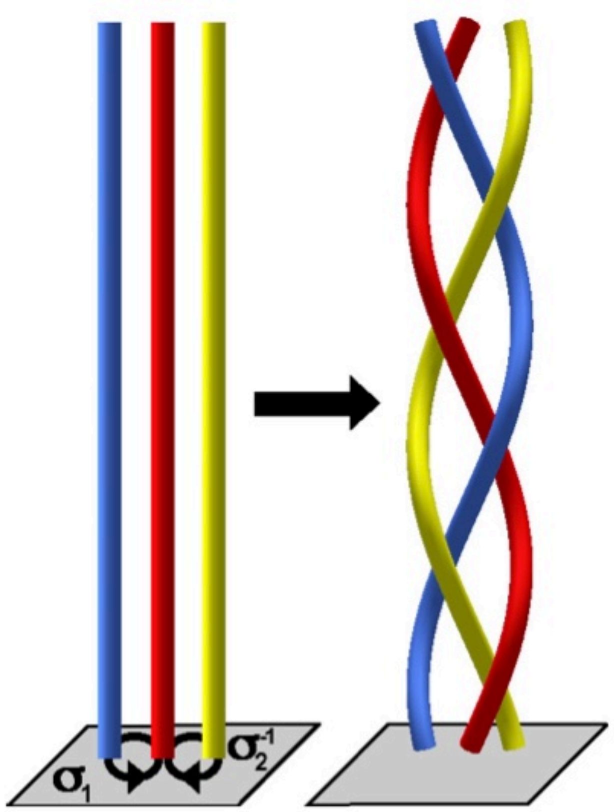


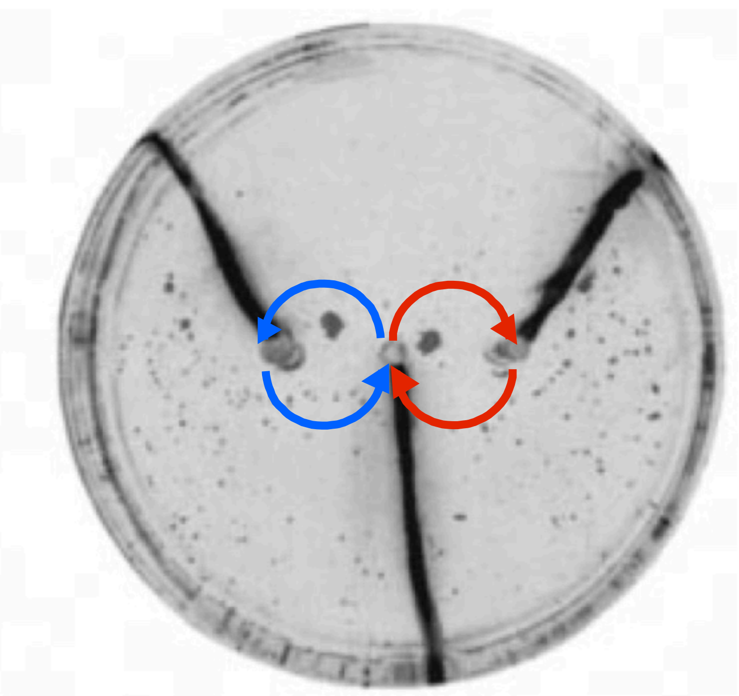



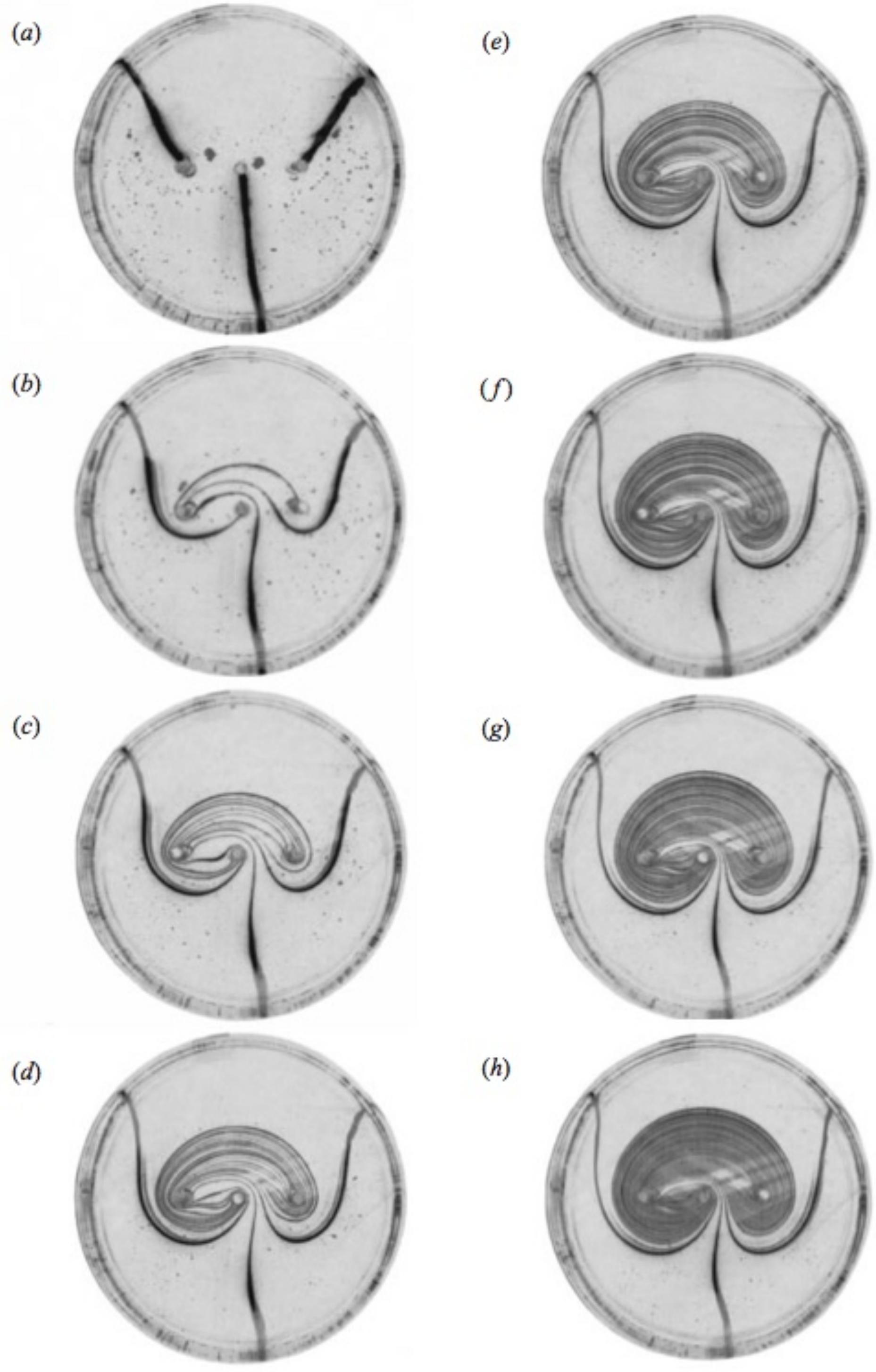

Various articles: AWS, G Hornig, DI Pontin, AR Yeates

# Why small scales?

- Magnetic braids equivalent to stirring
- Compare with stirring protocols
- and taffy pulling!






Boyland, Aref & Stremler, J. Fluid Mech., 403 277 (2000)

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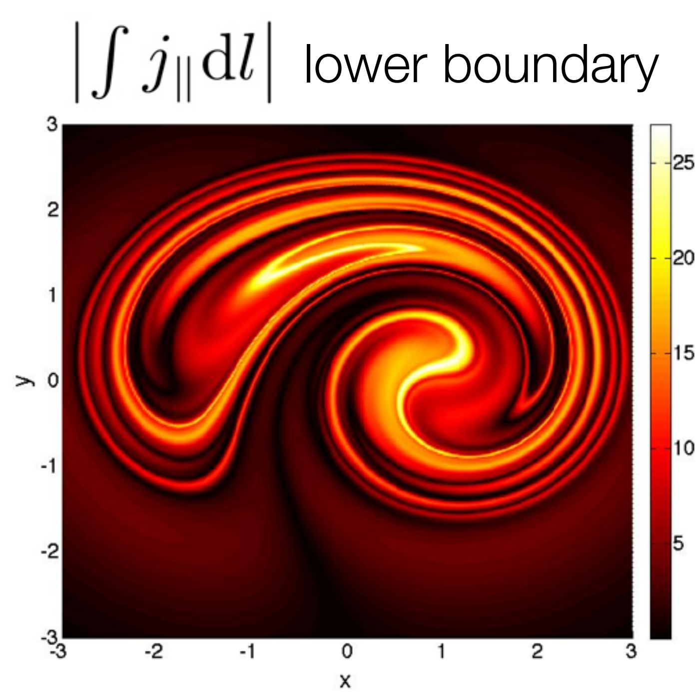
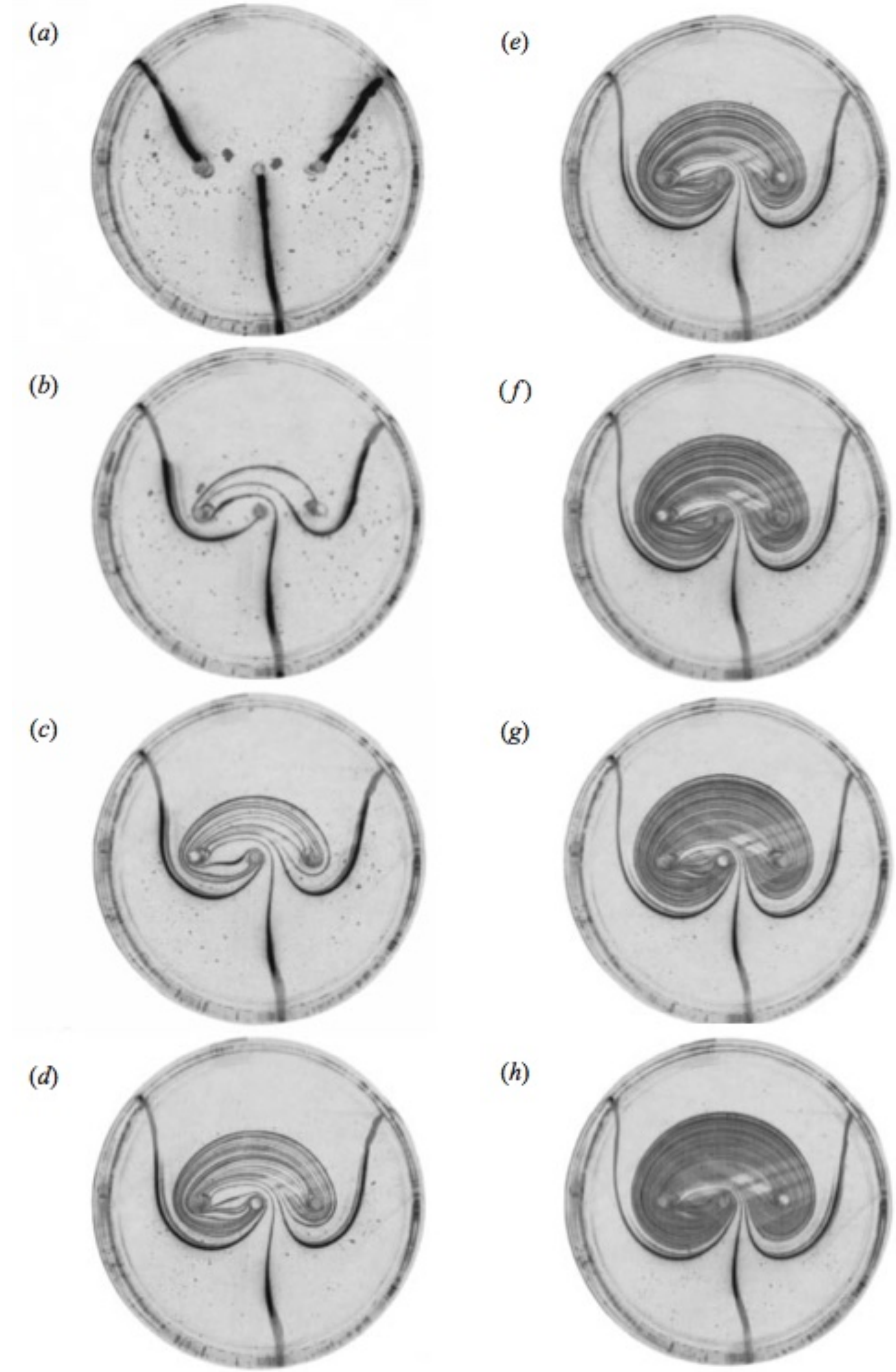
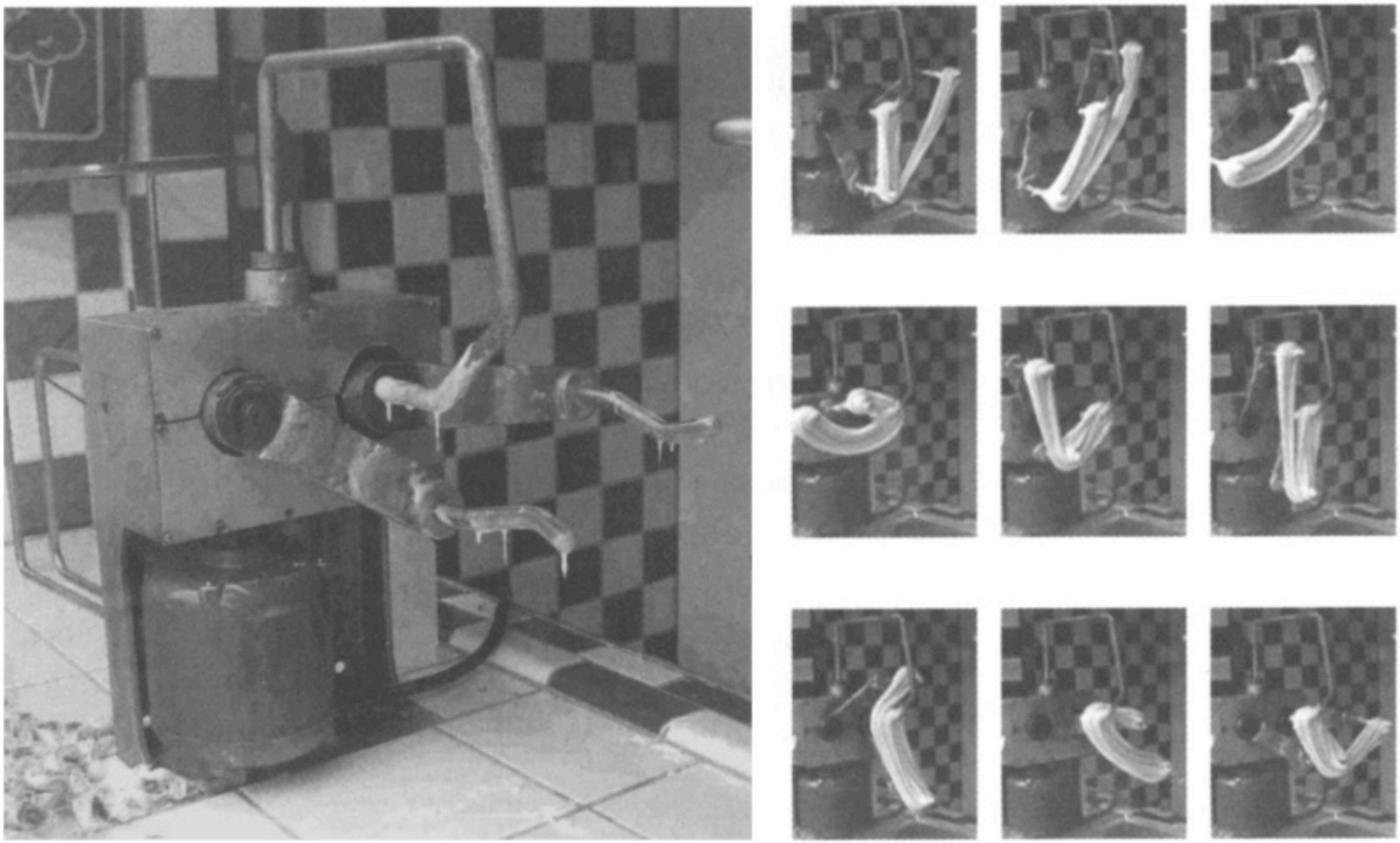



Boyland, Aref & Stremler, J. Fluid Mech., 403 277 (2000)

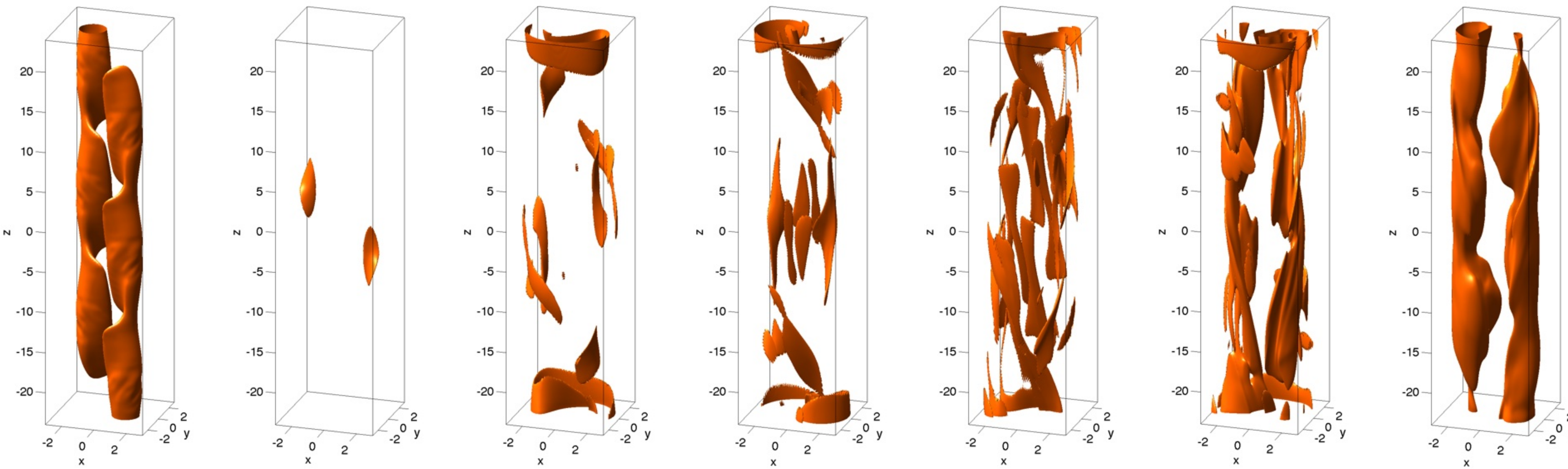


(a) (b)

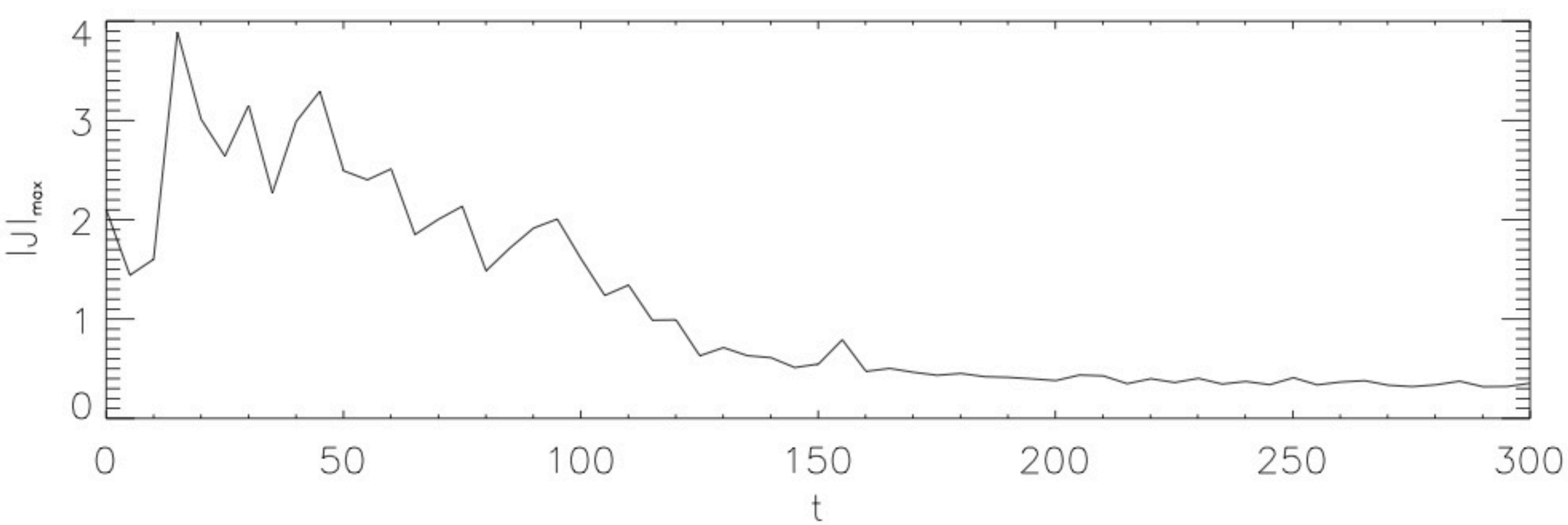
**Fig. 1.2** (a) *A taffy-pulling device.* (b) *Snapshots over a full period of operation, with taffy.*

MD Finn & JL Thiffeault, Topological Optimisation of Rod-Stirring Devices, SIAM Review, **53** 4 (2011)

# Non-ideal evolution



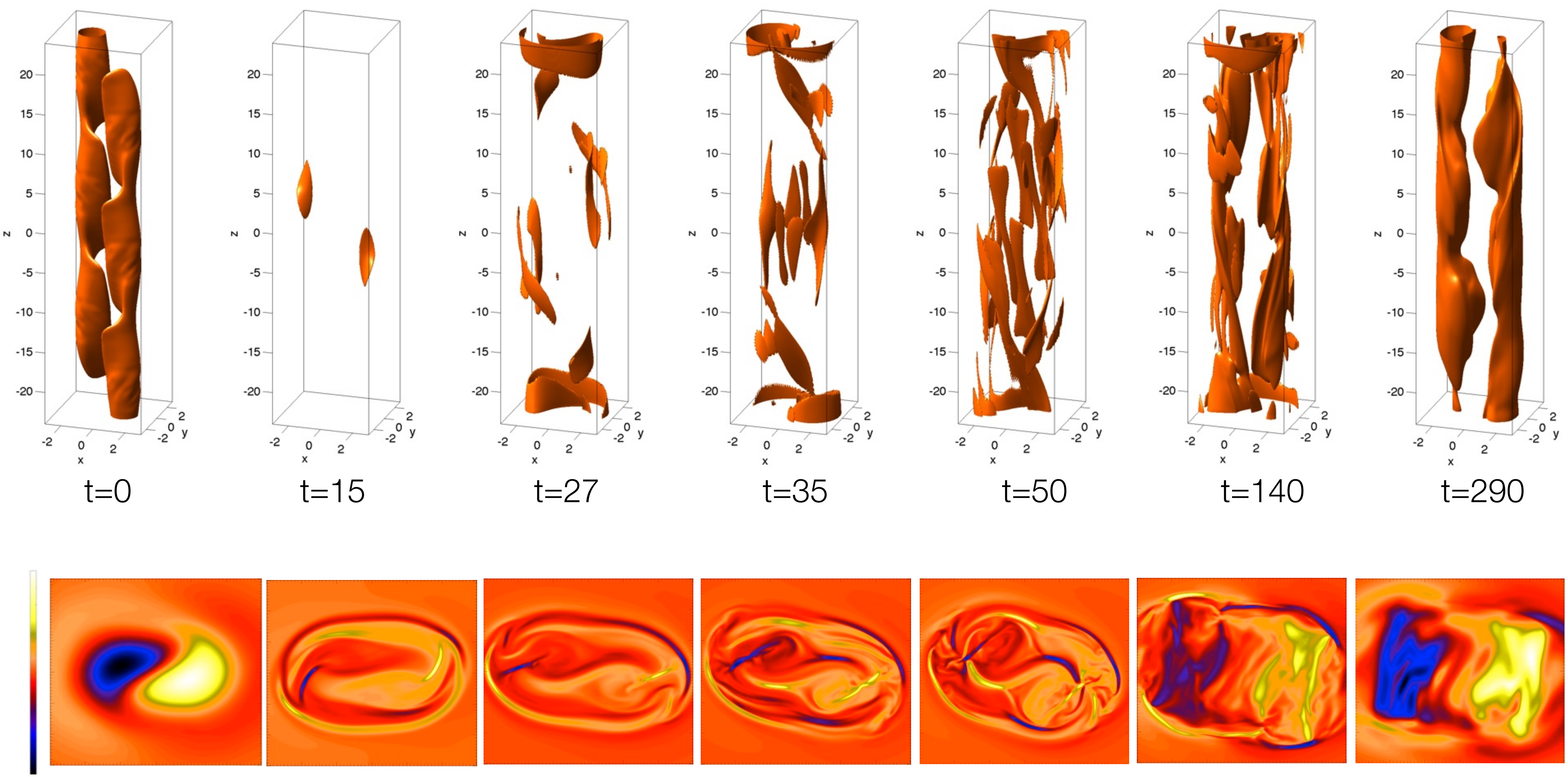

t=0 t=15 t=27 t=35 t=50 t=140 t=290




↑ Isosurfaces of current $|\mathbf{J}| = |\mathbf{J}|_{max}/2$.

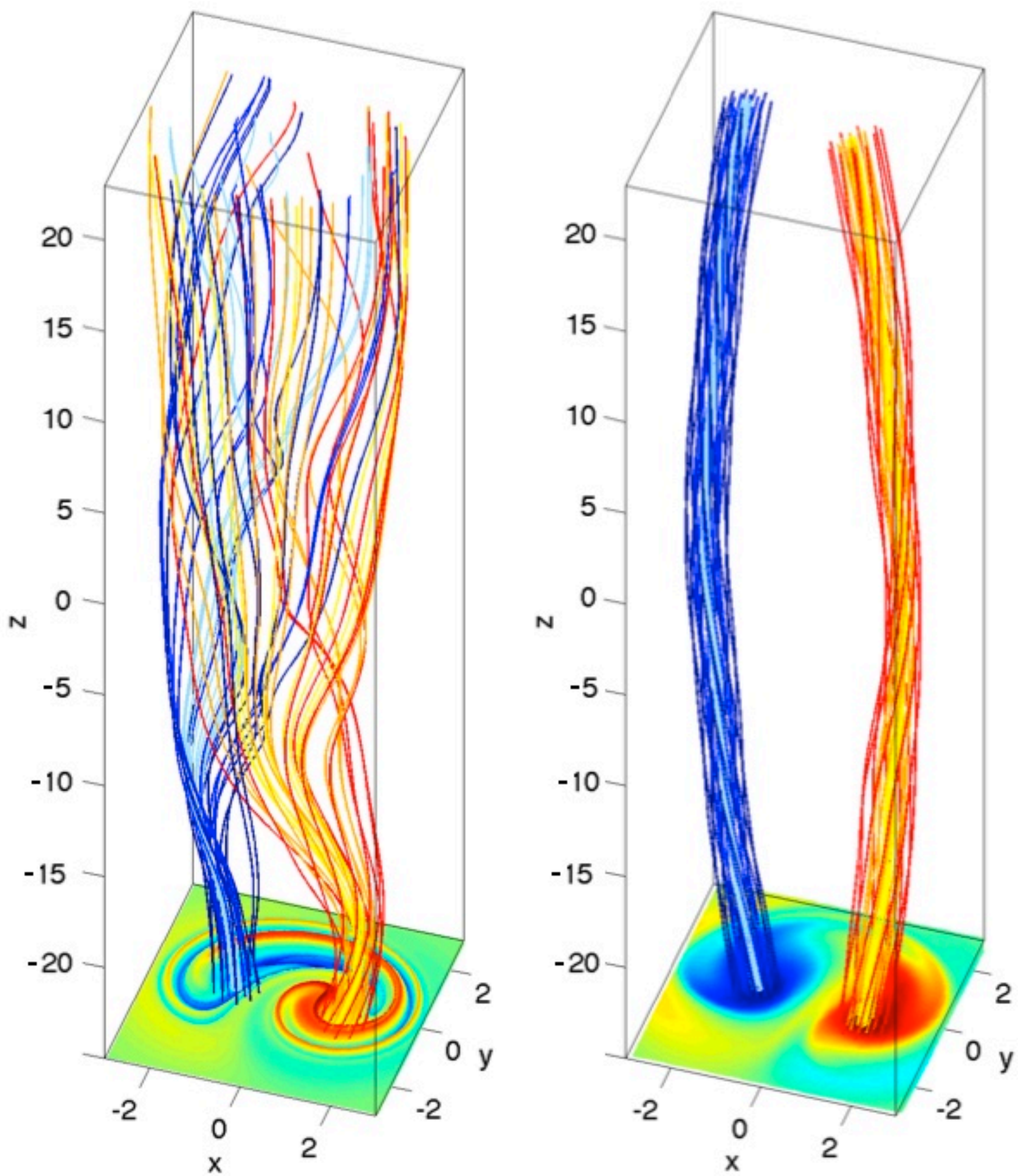
← Maximum $|\mathbf{J}|$ in domain with time.

# Non-ideal evolution




$J_z$, central plane

# Resistive relaxation: end state




- Magnetic field "unbraids", releasing energy
- Less energy release than would have been expected in standard theory
- Development of mathematical constraints on energy release