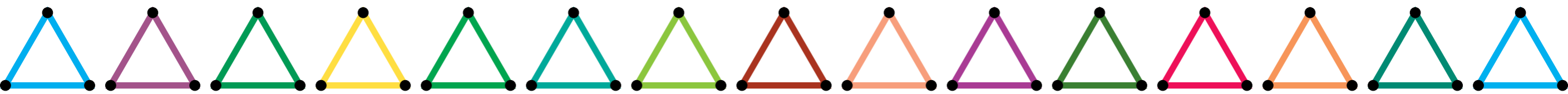


MATHEMATICAL JIGSAWS

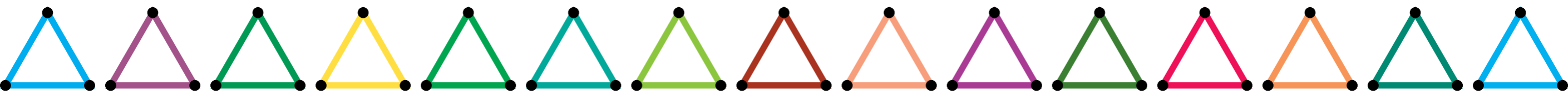
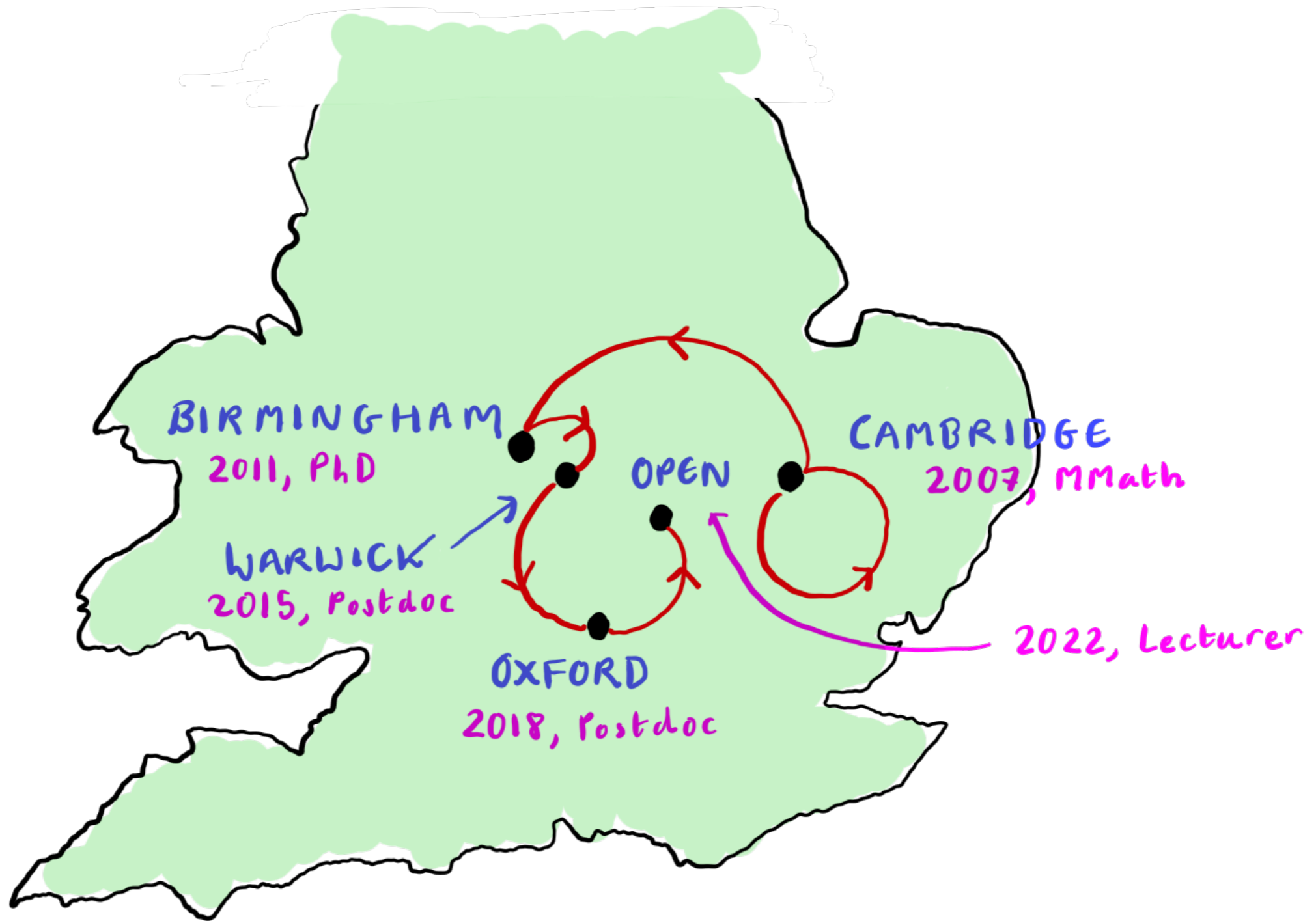
Experiences of a combinatorialist

PiWORKS seminar
26th March 2024

KATHERINE STADEN
THE OPEN UNIVERSITY



MY MATHEMATICAL JOURNEY



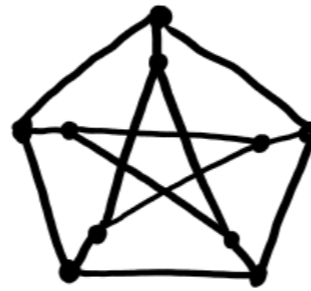
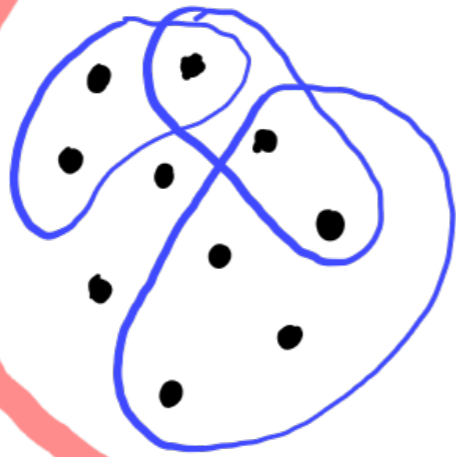
COMBINATORICS & DISCRETE MATHS

ENUMERATION

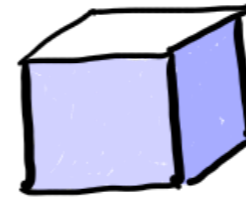
EXISTENCE

DISCRETE OBJECTS

RANDOM
OBJECTS



0110101110...

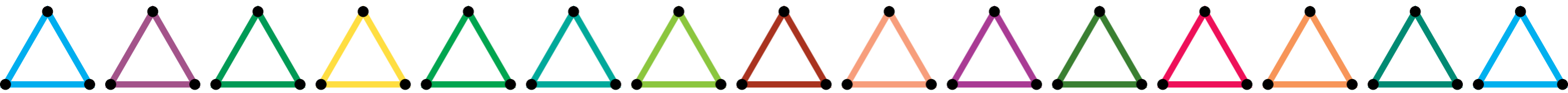


ALGORITHMS

{1, 2, 3, 4, 5, ...}

CONSTRUCTION

EXTREME
BEHAVIOUR



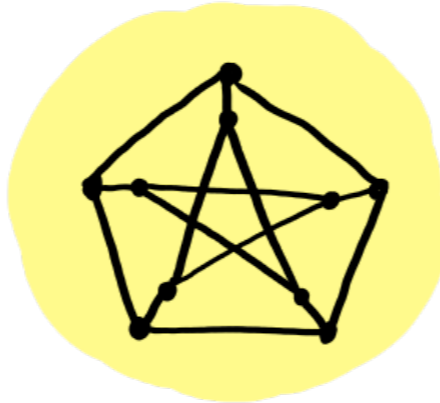
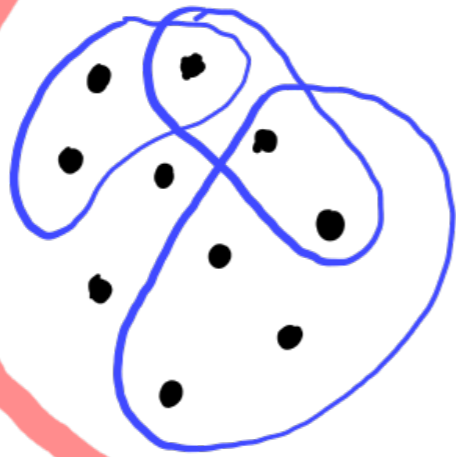
COMBINATORICS & DISCRETE MATHS

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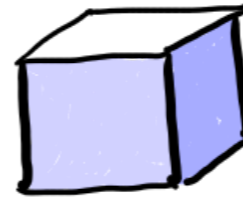
EXISTENCE

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ALGORITHMS

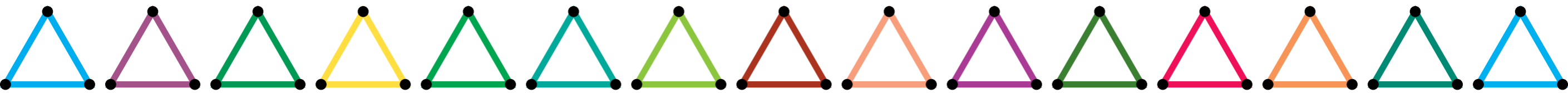
$\{1, 2, 3, 4, 5, \dots\}$

CONSTRUCTION

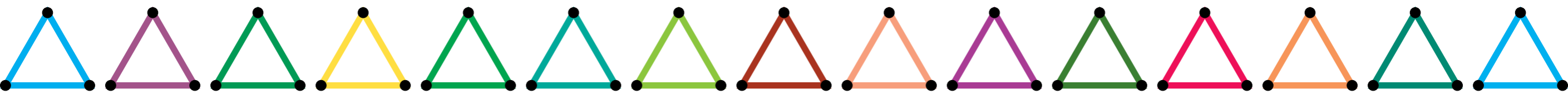
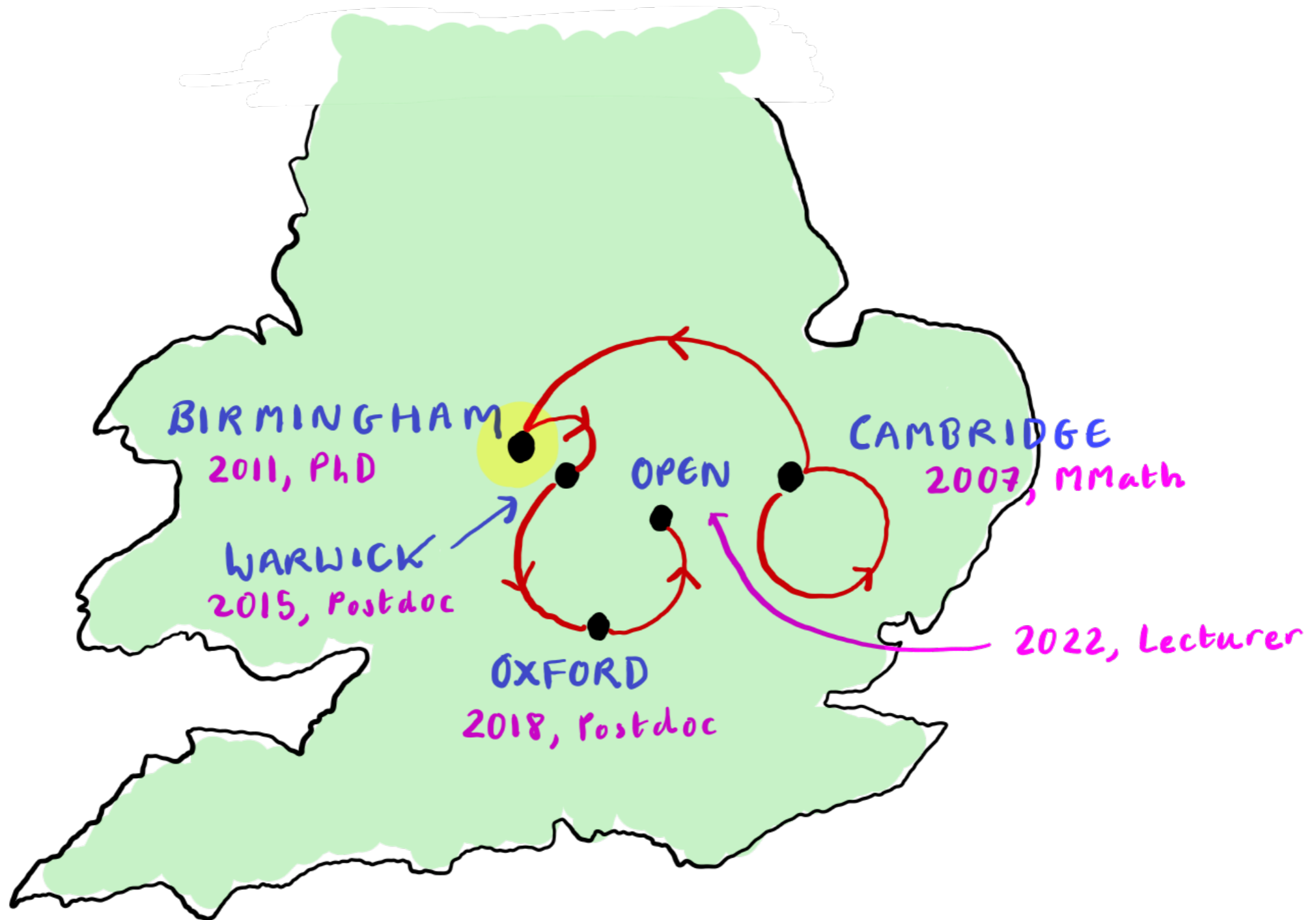
EXTREME
BEHAVIOUR

GRAPHS / NETWORKS

COMPUTER SCIENCE, BIOLOGY, PHYSICS, ...



MY MATHEMATICAL JOURNEY

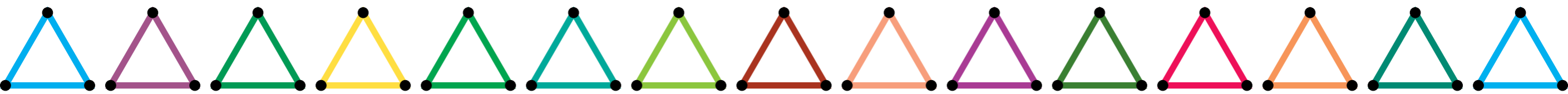
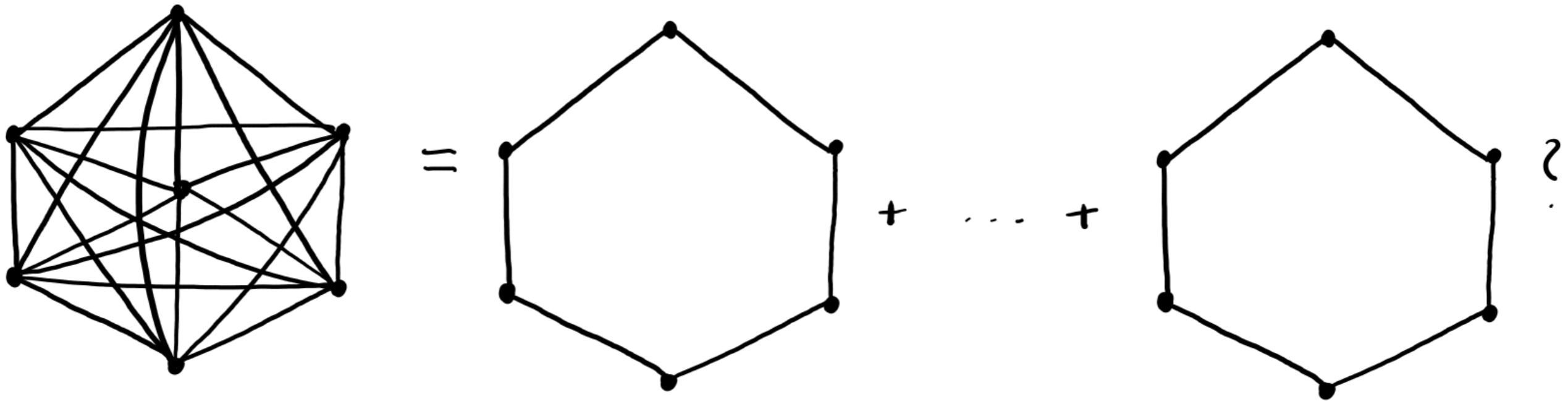


DECOMPOSITIONS IN BIRMINGHAM

A mathematical jigsaw: given a discrete object and some puzzle pieces, is it possible to make the object from these pieces?

COMPLETE GRAPH INTO HAMILTON CYCLES

Walecki 1892

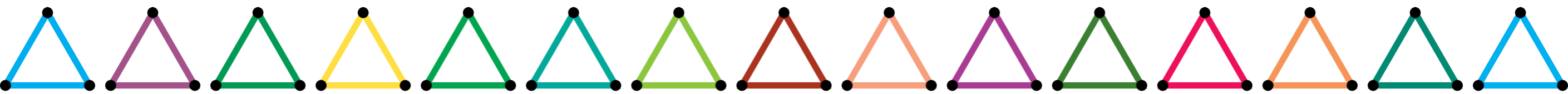
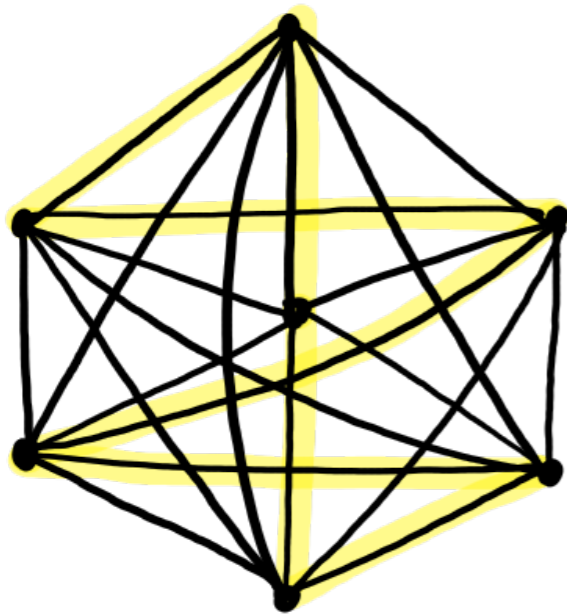


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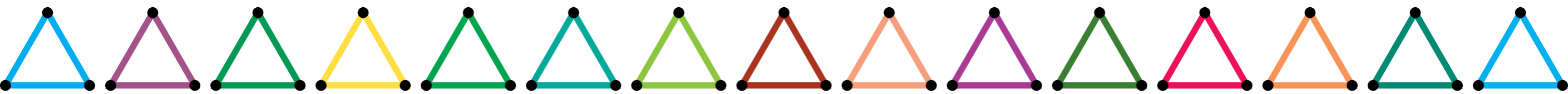
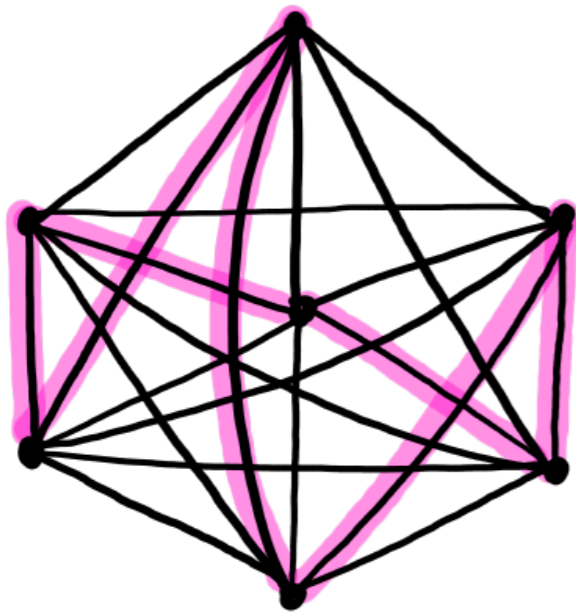


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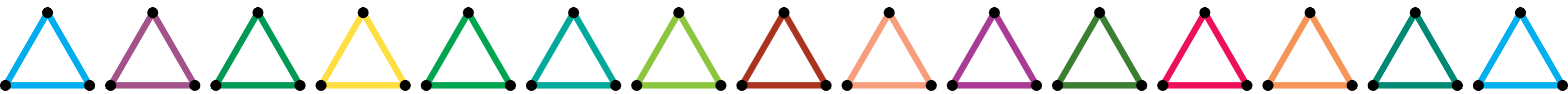
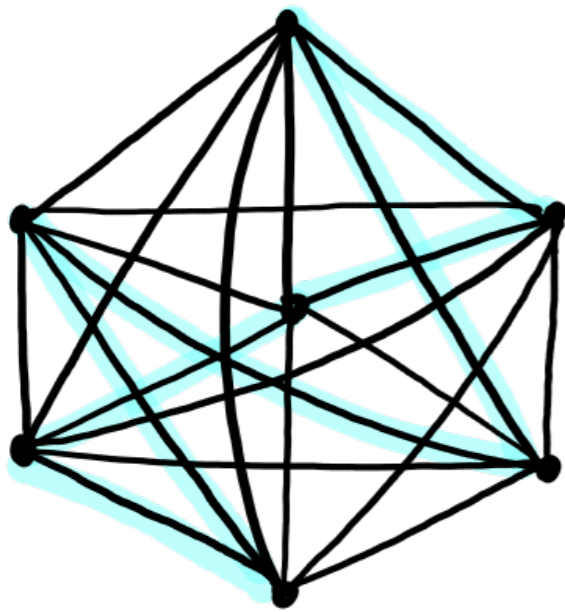


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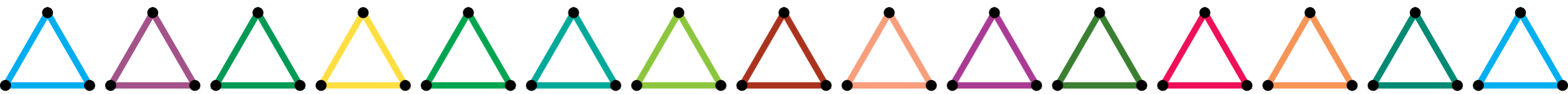
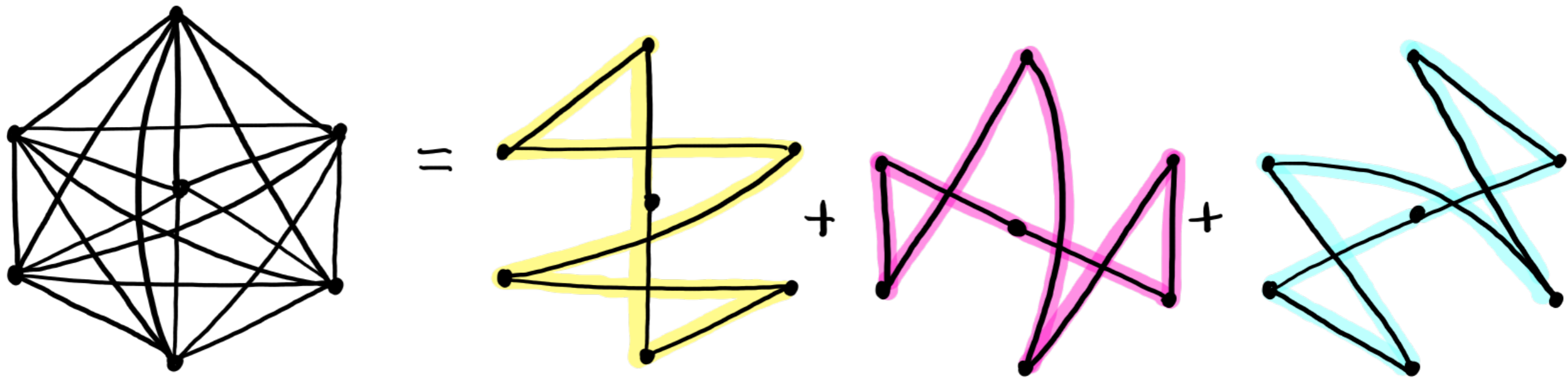


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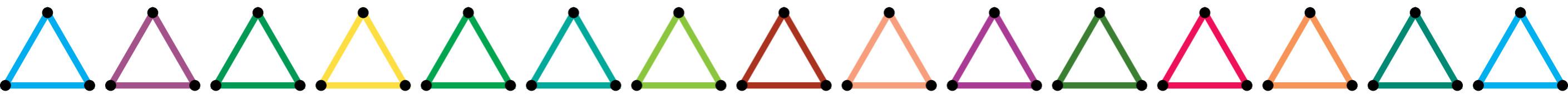
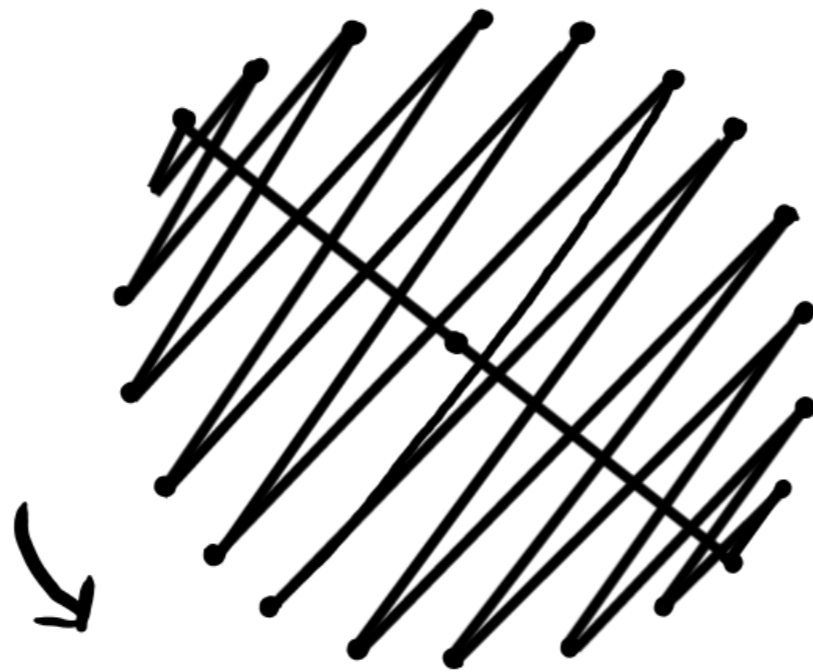


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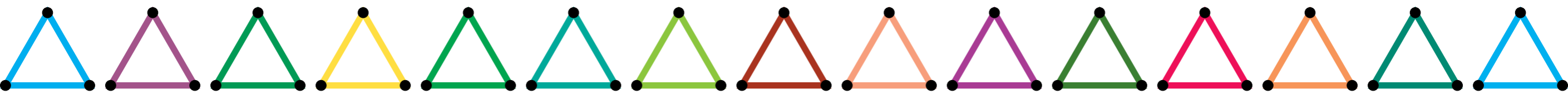
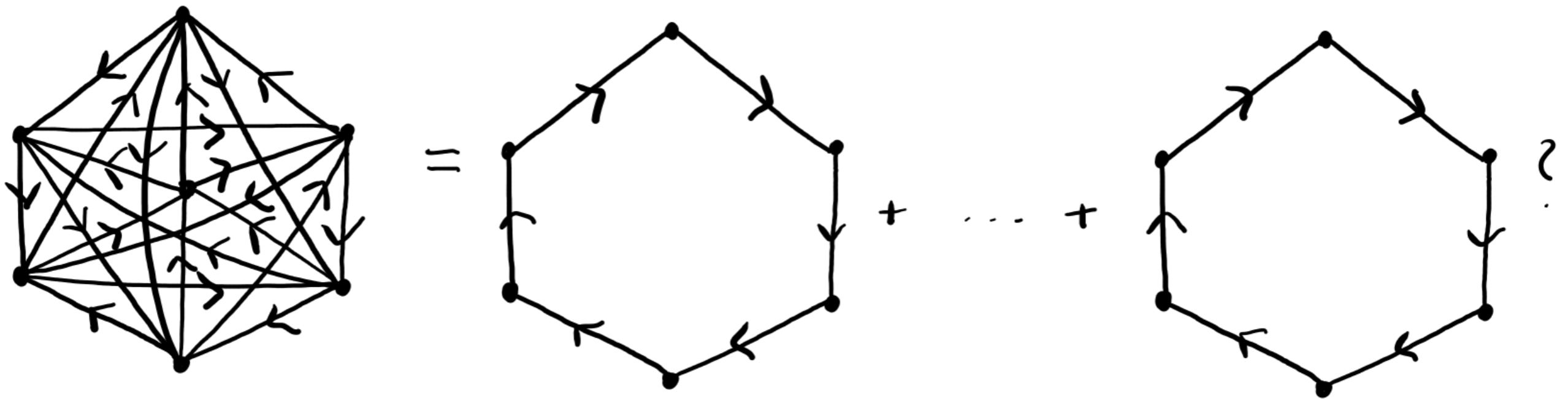


DECOMPOSITIONS IN BIRMINGHAM

A mathematical jigsaw: given a discrete object and some puzzle pieces, is it possible to make the object from these pieces?

REGULAR TOURNAMENT INTO HAMILTON CYCLES

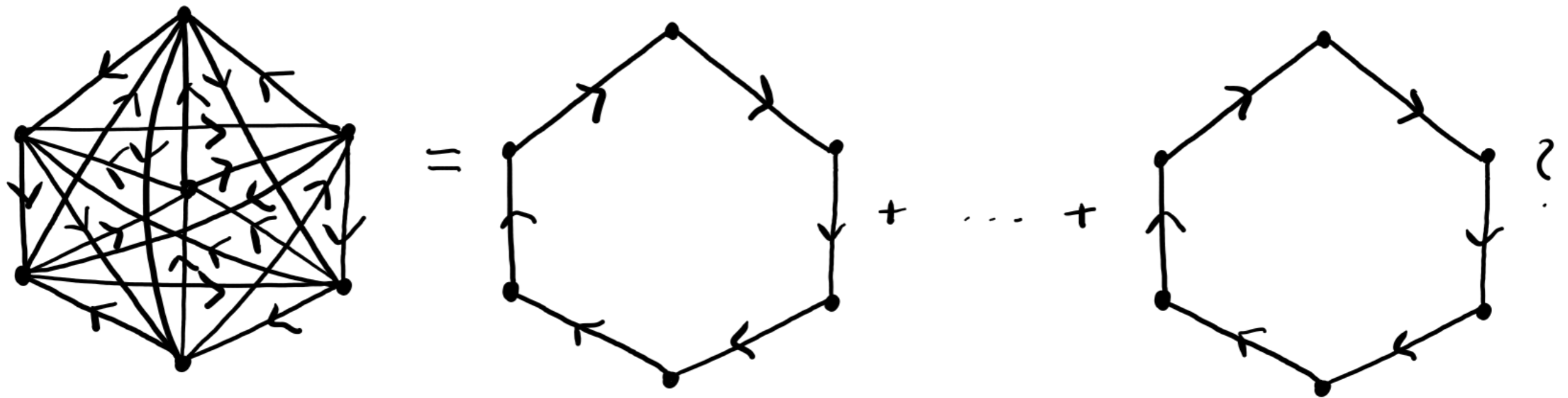
Kelly's conjecture 1968



DECOMPOSITIONS IN BIRMINGHAM

REGULAR TOURNAMENT INTO HAMILTON CYCLES

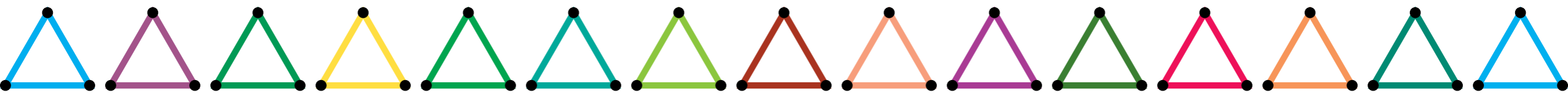
Kelly's conjecture 1968



My first project: approximate version with *Osthus 2013*

used in

Proof of conjecture for large graphs *Kühn-Osthus 2013*



DECOMPOSITIONS IN BIRMINGHAM

REGULAR TOURNAMENT INTO HAMILTON CYCLES

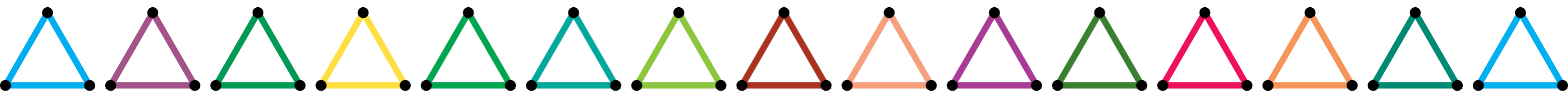
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↓
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Mathematics: approximating large graphs by finite/continuous objects Szemerédi 1970s (Abel prize)



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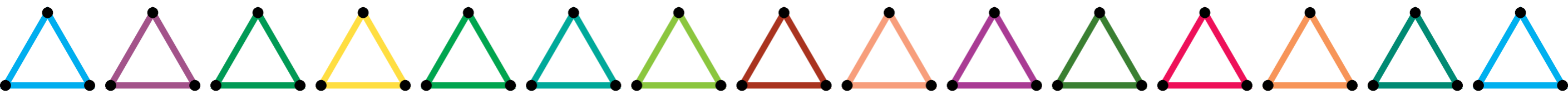
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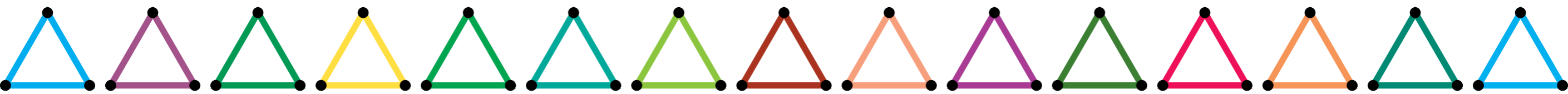
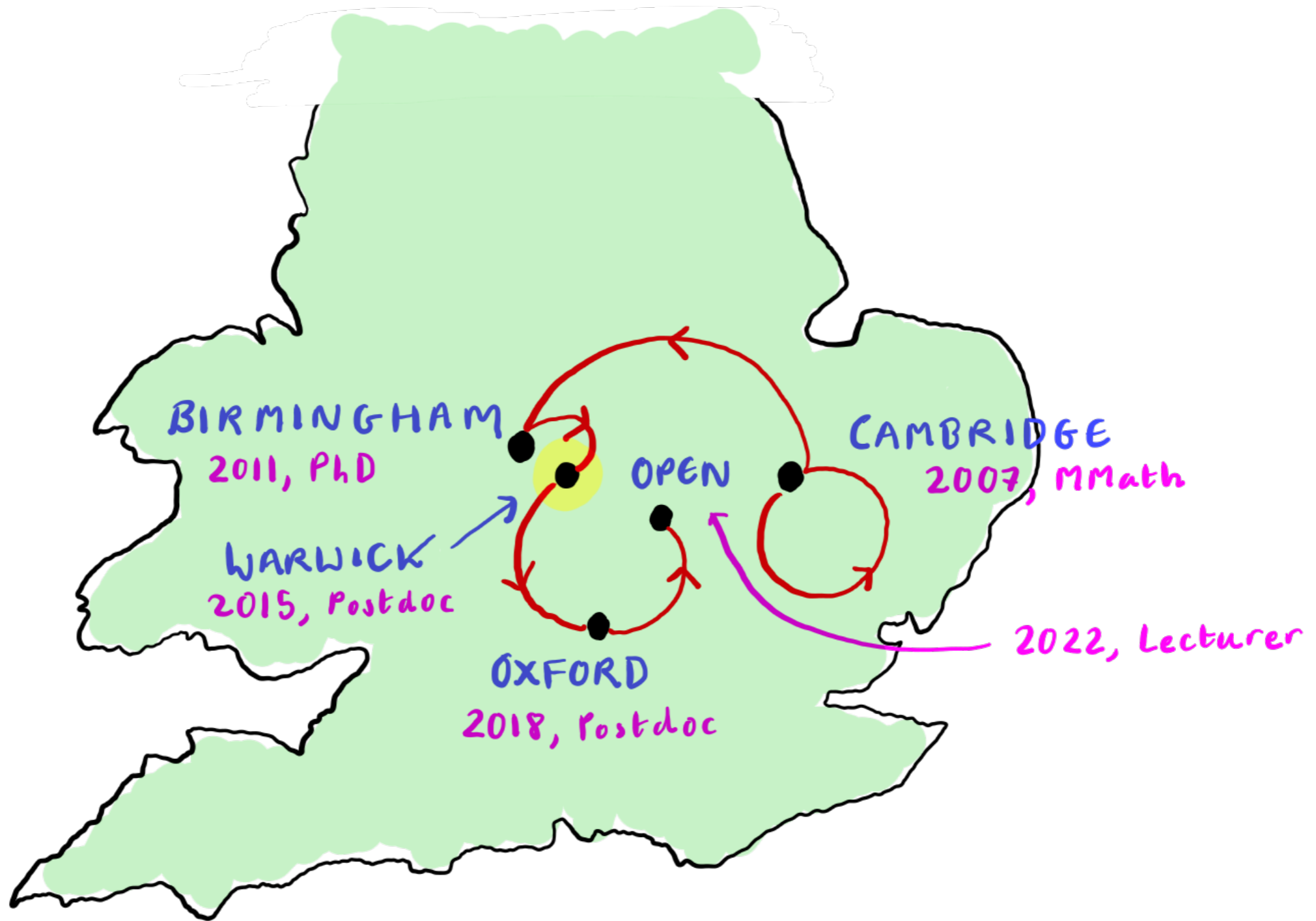
↓
Proof of conjecture for large graphs Kühn-Osthus 2013

Mathematics: approximating large graphs by finite/continuous objects Szemerédi 1970s (Abel prize)

Experiences: no 'big' ideas - small ideas are important too
many small parts make the whole

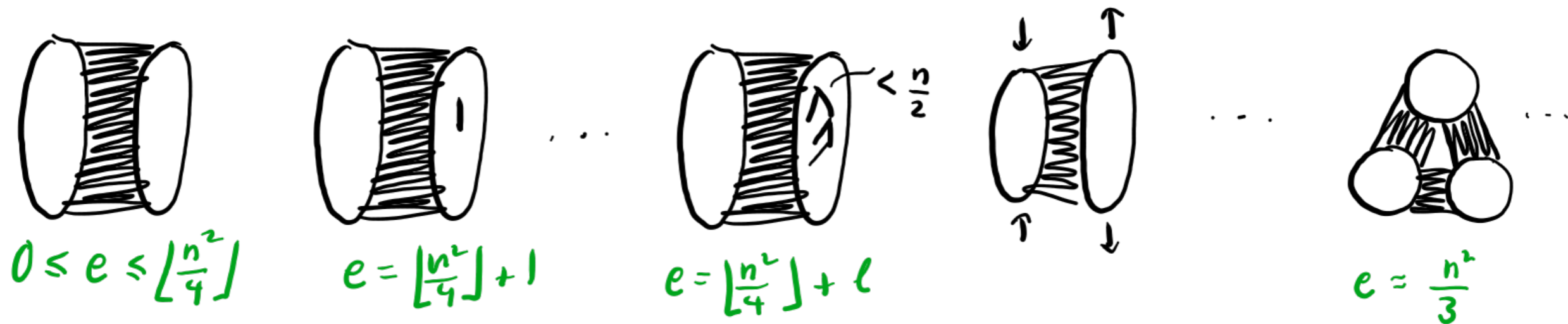


MY MATHEMATICAL JOURNEY



COUNTING TRIANGLES IN WARWICK

What is the minimum number of triangles in a graph with a given number of vertices and edges? Erdős-Rademacher problem



$$0 \leq e \leq \lfloor \frac{n^2}{4} \rfloor$$

$$e = \lfloor \frac{n^2}{4} \rfloor + 1$$

$$e = \lfloor \frac{n^2}{4} \rfloor + l$$

$$l < \frac{n}{2}$$

$$e = \frac{n^2}{3}$$

$$\Delta = 0$$

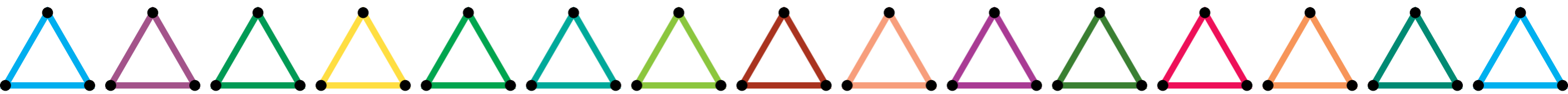
Mantel
1907

$$\Delta = \lfloor \frac{n}{2} \rfloor$$

Rademacher
1941

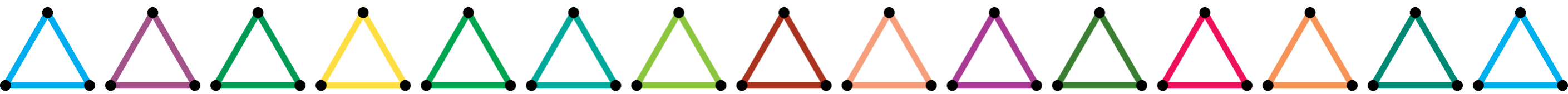
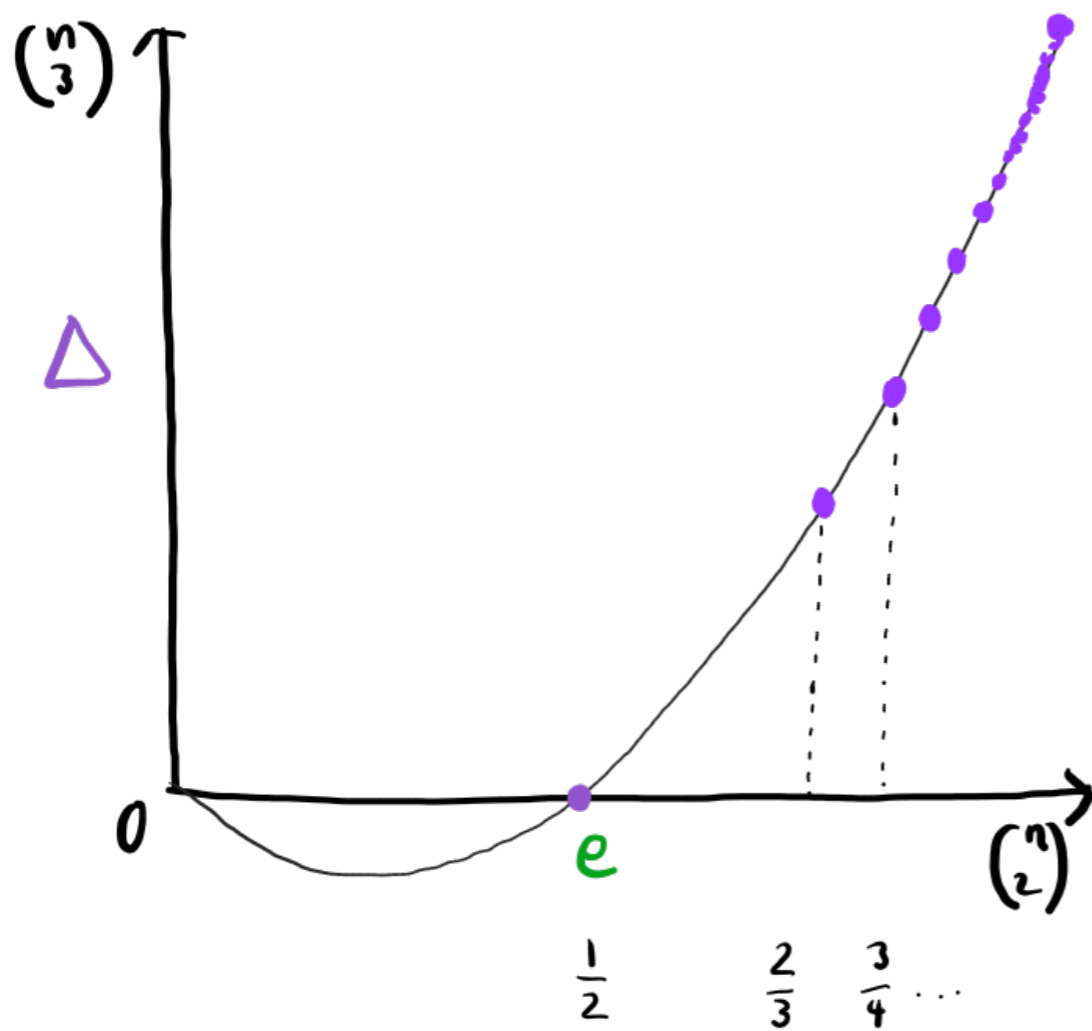
$$\Delta = l \lfloor \frac{n}{2} \rfloor$$

Conjecture: Erdős 1950s/60s
Lovász-Simonovits 1976



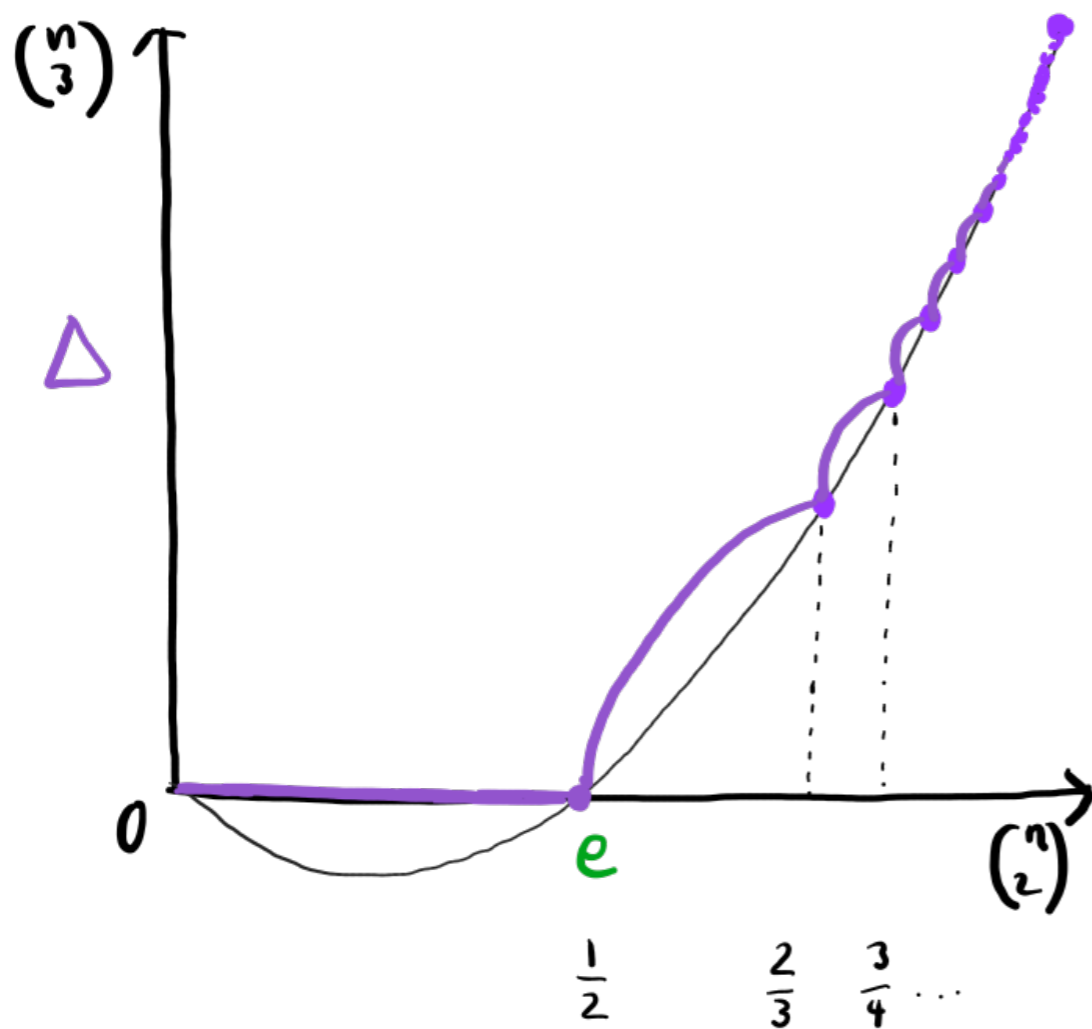
COUNTING TRIANGLES IN WARWICK

What is the minimum number of triangles in a graph with a given number of vertices and edges?



COUNTING TRIANGLES IN WARWICK

What is the minimum number of triangles in a graph with a given number of vertices and edges?



Mantel 1907

Rademacher 1941

Erdős 1955, 1962, 1967

Goodman 1959

Bollobás 1976

Lovász-Simonovits 1976

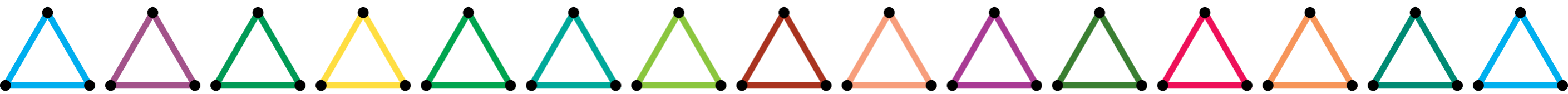
Fisher 1989

Razborov 2008

Pikhurko-Razborov 2013

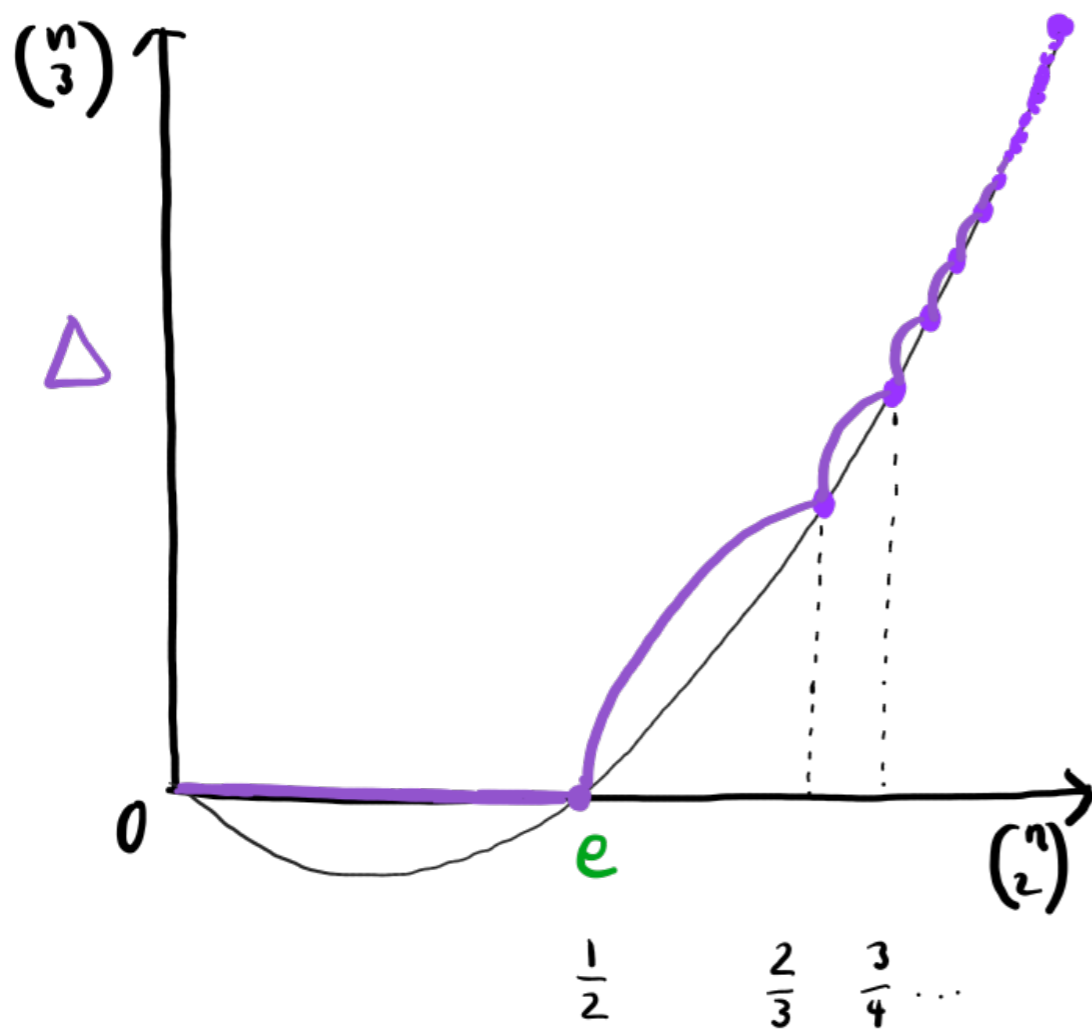
Liu-Pikhurko-S. 2016-2020

?



COUNTING TRIANGLES IN WARWICK

What is the minimum number of triangles in a graph with a given number of vertices and edges?



Razborov
all e , approximate

Liu-Pikhurko-S.
 $0 \leq e \leq 0.99 \binom{n}{2}$

Mathematics:

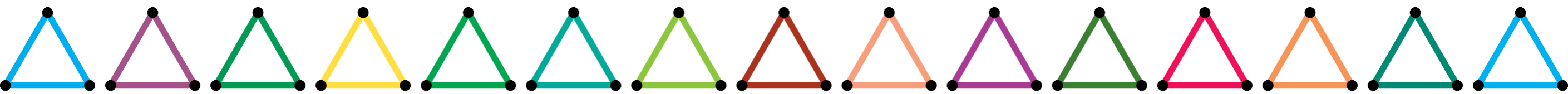
R flag algebras ← revolutionary

LPS stability argument

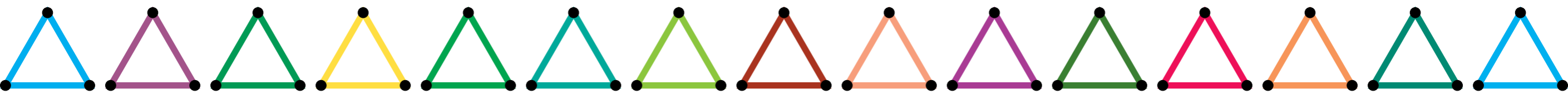
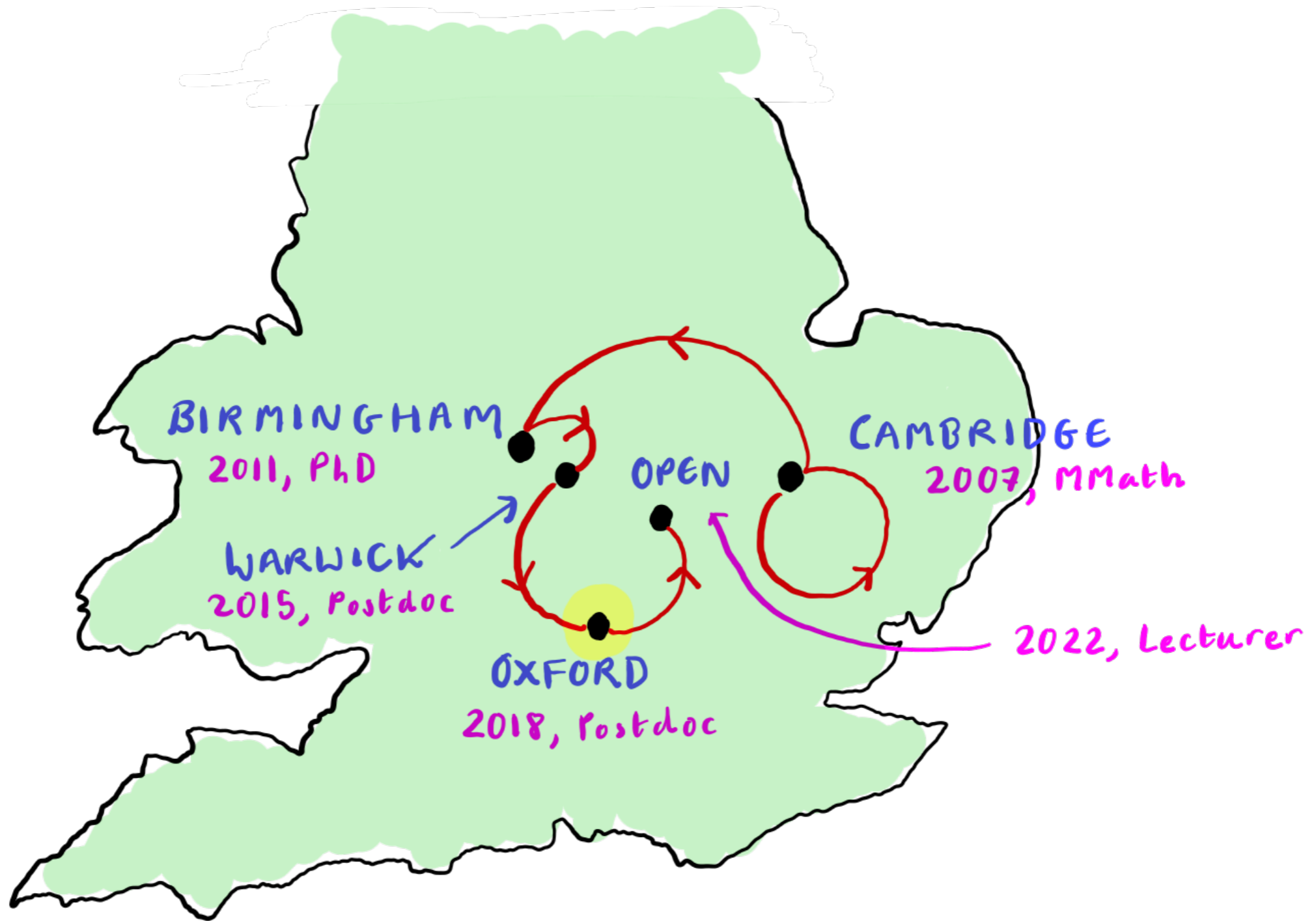
Few imperfections → no imperfections

Experiences: obsession!

long project (proof + time)



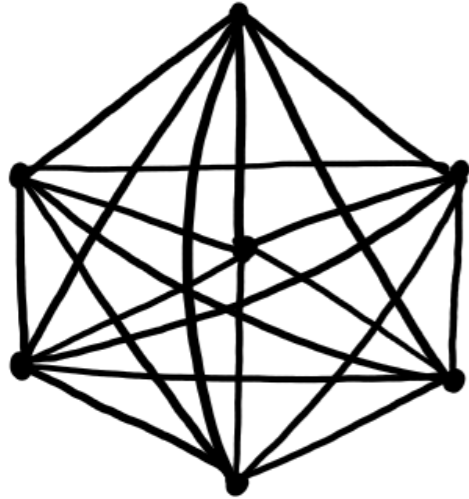
MY MATHEMATICAL JOURNEY



MORE DECOMPOSITIONS IN OXFORD

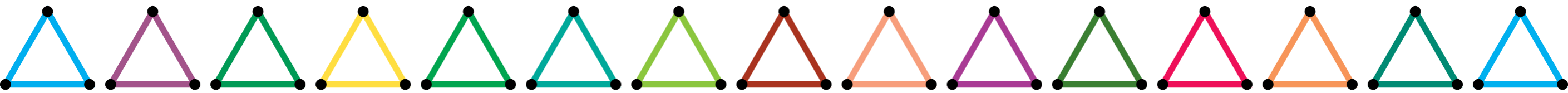
EXISTENCE OF DESIGNS

Woolhouse 1844, Kirkman 1847, Steiner 1853 conjectures



$$= \triangle + \triangle + \triangle + \triangle + \triangle + \triangle + \triangle \quad ?$$

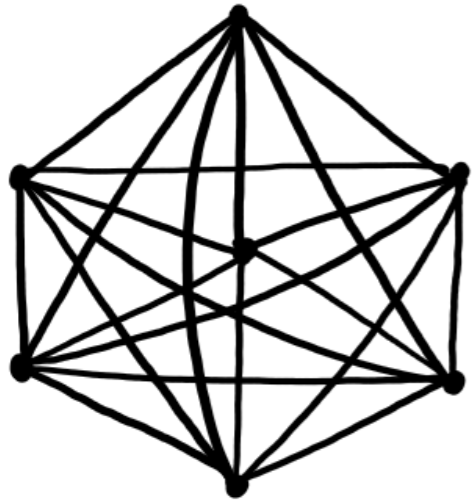
Wilson 70s, Keevash 2014



MORE DECOMPOSITIONS IN OXFORD

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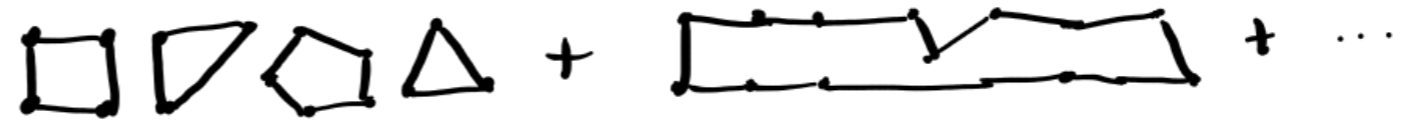


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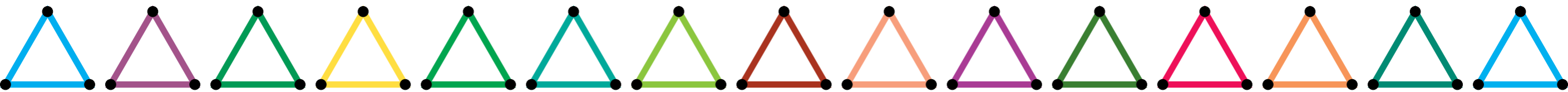
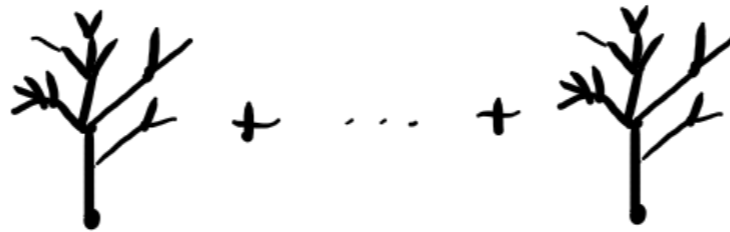
OBERWOLFACH PROBLEM

Ringel 1967 conj.



RINGEL'S CONJECTURE

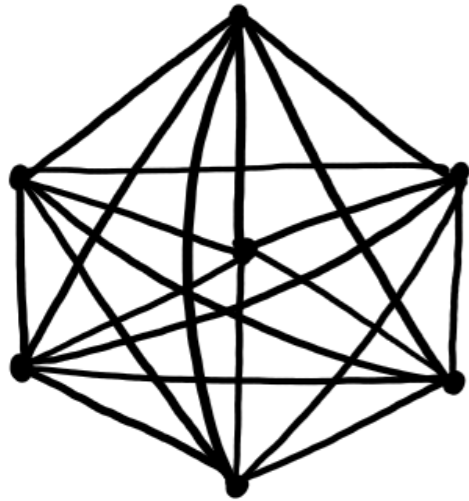
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MORE DECOMPOSITIONS IN OXFORD

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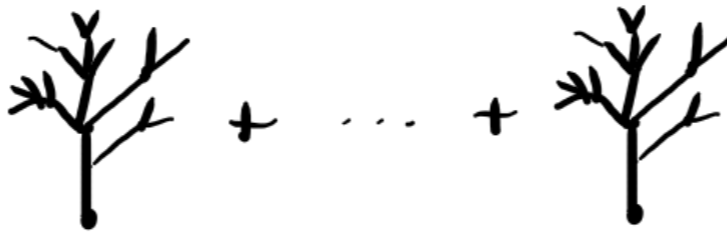
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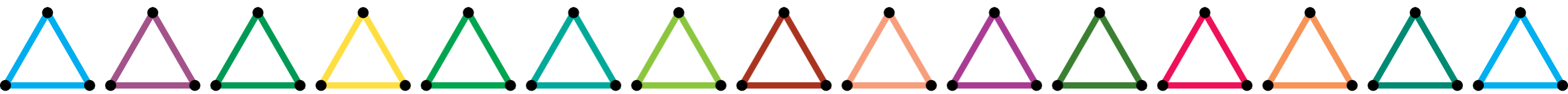
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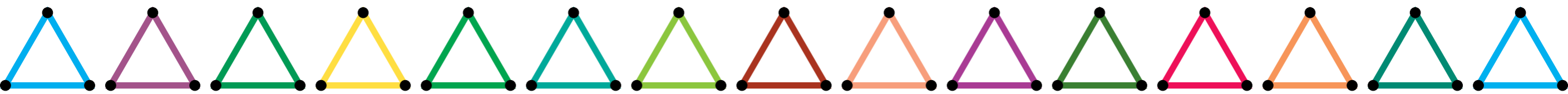
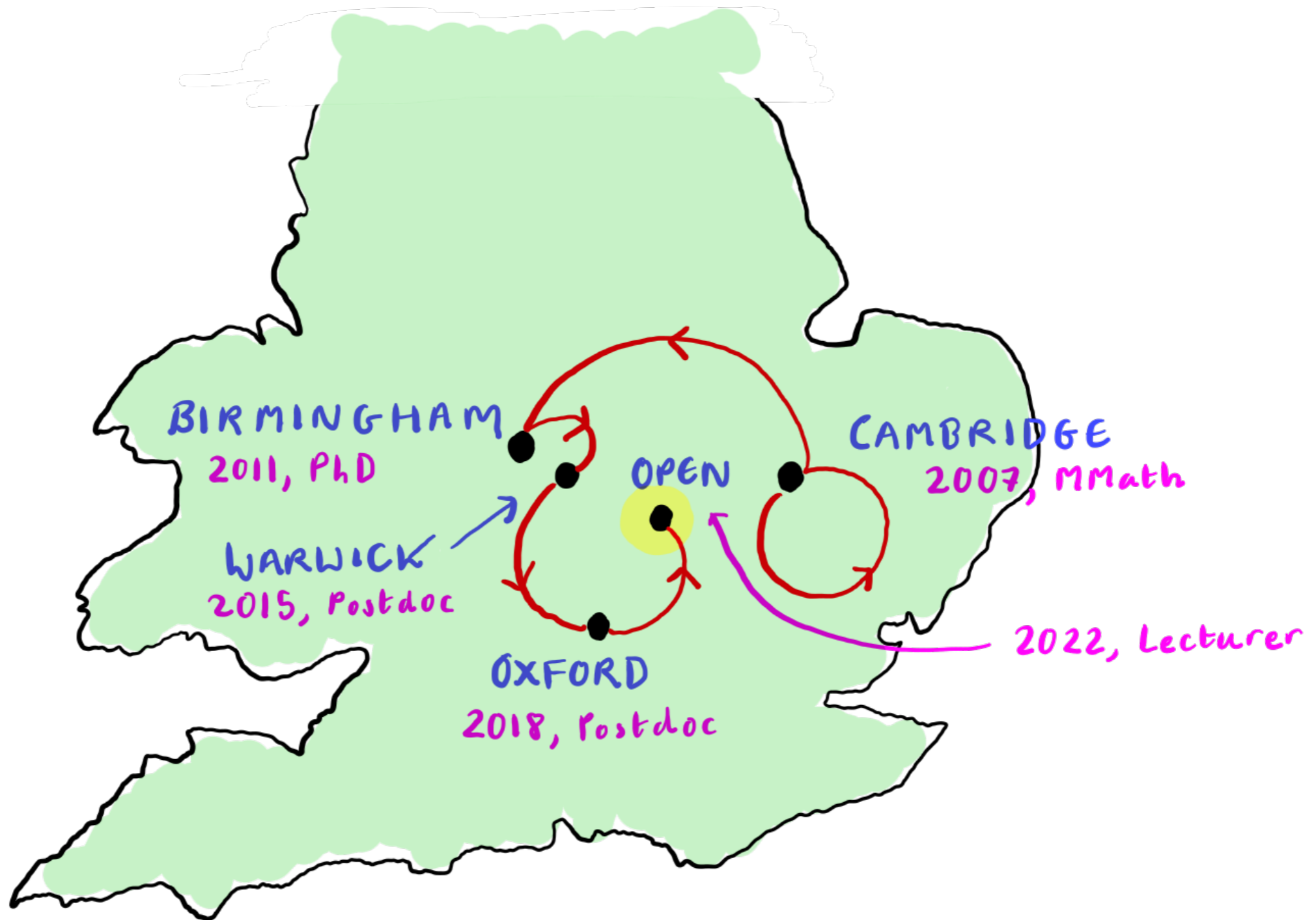


Mathematics: K New 'absorption' method
 KS Power of randomness

Experiences: Scoped in
 New research direction



MY MATHEMATICAL JOURNEY



DOING RESEARCH IN MATHEMATICS

THINKING
IN THE
RIGHT WAY

MOTIVATION

EXPERIENCE

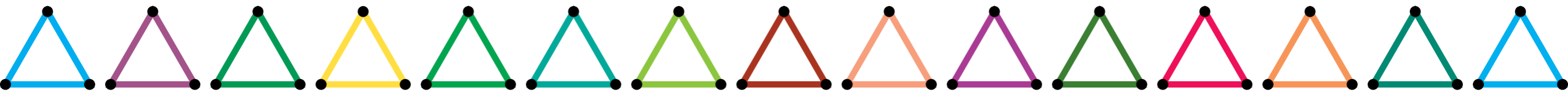
AMBITION

HARD
WORK

UNDERSTANDING

NATURAL
ABILITY

COLLABORATION
& COMMUNICATION



BEING A WOMAN IN MATHEMATICS

Most people want you to succeed

The situation has improved, but there are still challenges

My challenges

~~Feeling out of place~~

~~Instability / moving~~

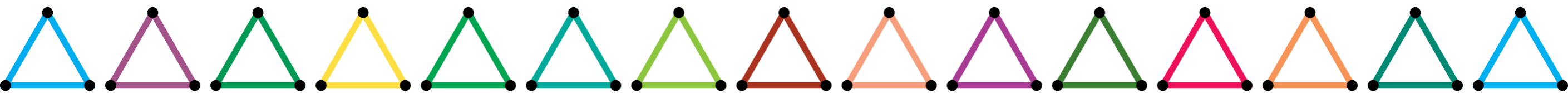
Other people's assumptions

Leadership / respect

career
ladder



Usually, I am just a person doing mathematics



BEING A WOMAN IN MATHEMATICS

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Thanks!

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